

Magnetism and Electromagnetic Induction

CHAPTER 3

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3

Permanent Magnets and Magnetic Properties of Matter

INTRODUCTION

In the previous chapter, we studied that electric currents produce magnetic field, i.e., electricity generates magnetism. The terms such as magnetic field lines, magnetic flux, magnetic dipole moment etc., were discussed in the context of magnetic fields due to current flowing through a straight conductor, a circular coil, a solenoid and a toroid. These terms are closely related to magnetism, a complete topic in its own right. It is therefore imperative that we look into the details of magnetism afresh by combining its relation to electric current.

The science of magnetism grew when people observed that a certain ore could attract bits of iron and always pointed in a particular direction when freely suspended.

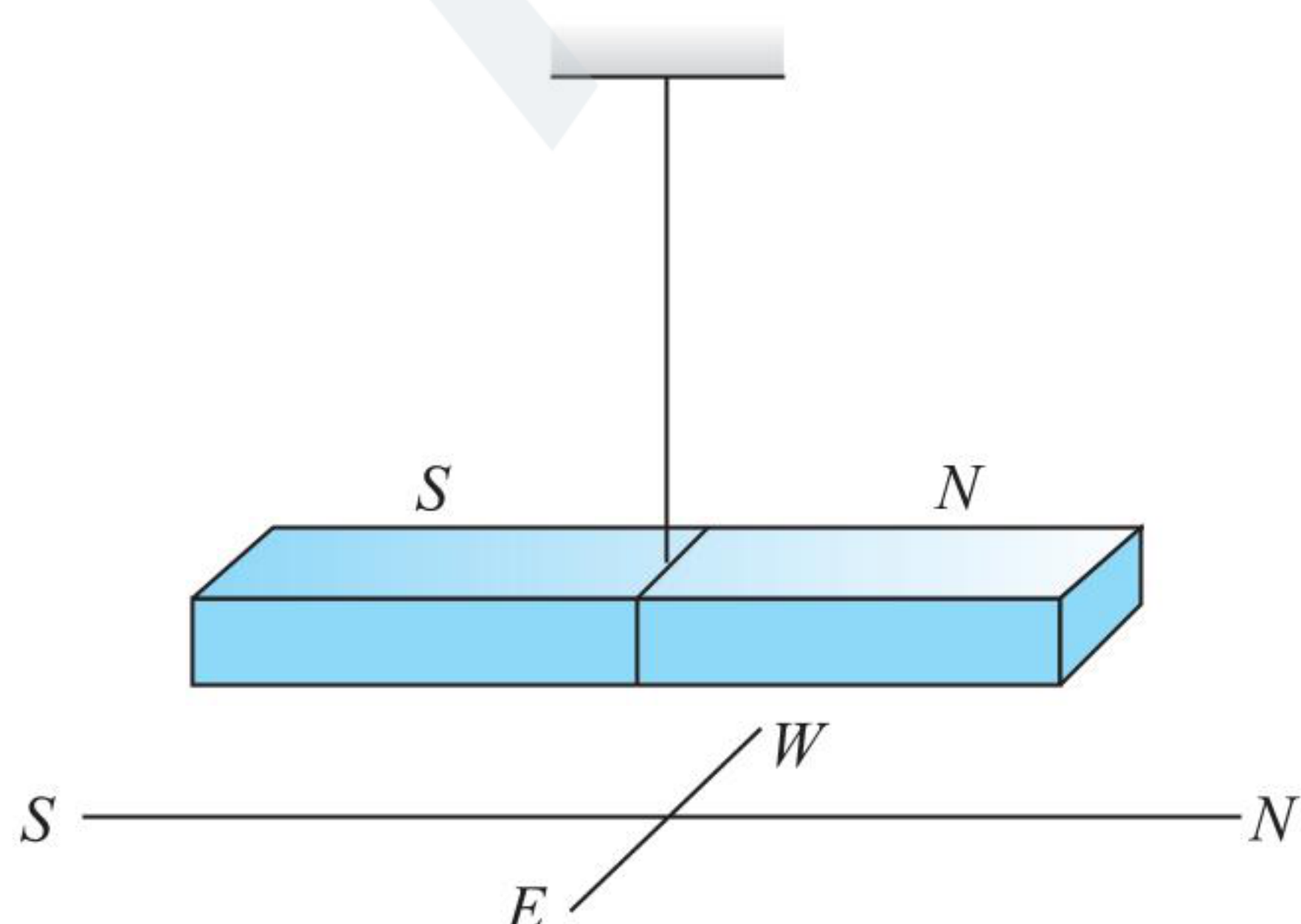
The property of attracting small pieces of iron and steel is referred to as **magnetism**. The mineral exercising this influence is called a **natural magnet**. Thus, a magnet is a substance which has attractive and directive properties. Natural magnet consists of a number of oxides of iron with Fe_3O_4 as its structural formula.

The natural magnets are weak and often irregular in shape. It is found that when natural magnets are rubbed against iron or steel bars, the same property of attraction is communicated to these pieces. They are, therefore, called **artificial magnets**. Though the magnets we commonly use are artificial magnets, which can be of desired shape and strength.

BAR MAGNET

A bar magnet is a rod shaped metal object which produces magnetic field in its surrounding at the two ends of it, there are two magnetic poles: north pole and south pole.

A bar magnet attracts iron fillings near the ends. A magnet when suspended freely with the help of a thread always rests in the **north-south** direction. The pole pointing towards north is called **north pole** and the one pointing towards south is called **south pole**.



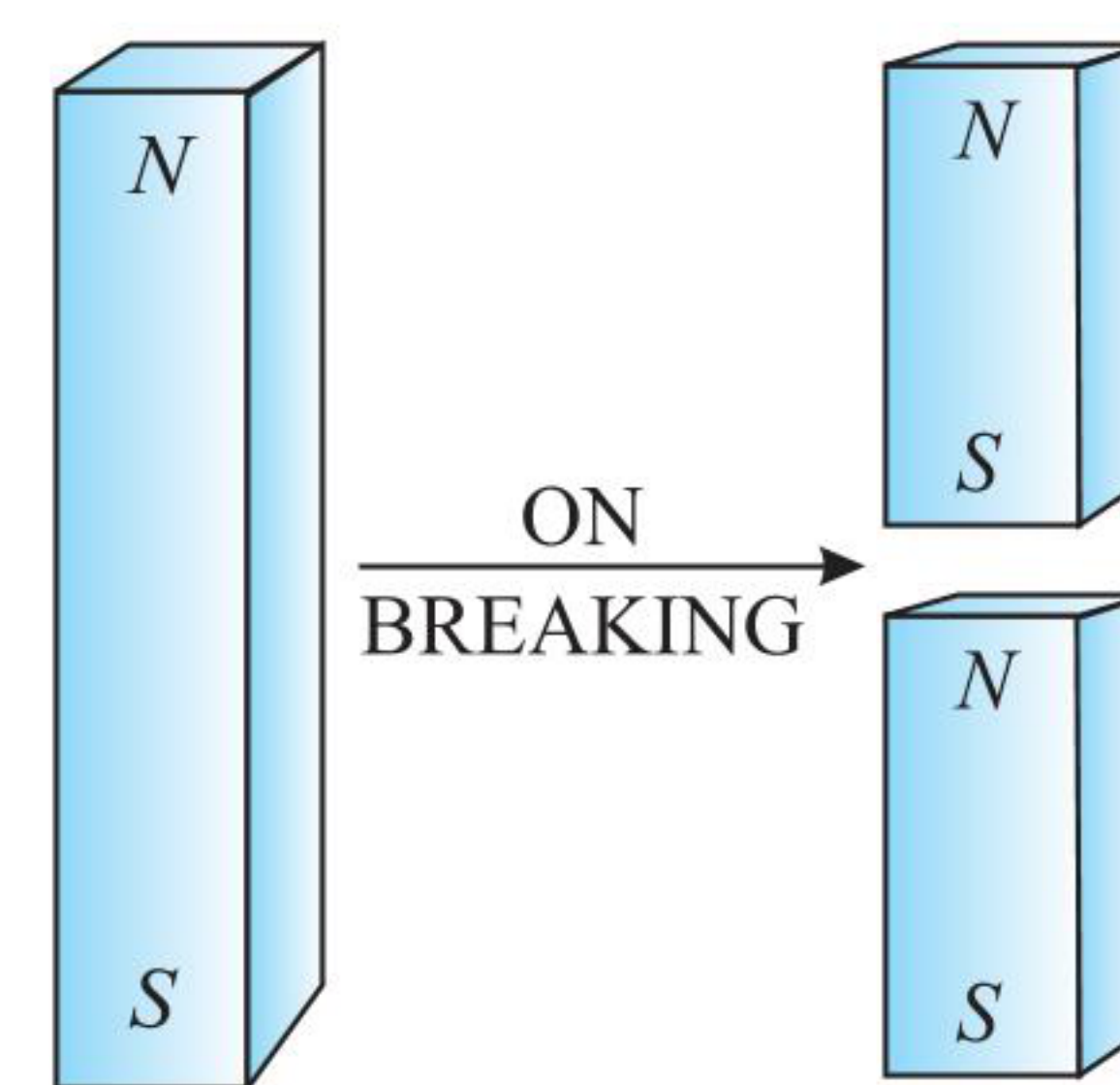
PROPERTIES OF BAR MAGNET

A bar magnet has the following properties:

- A magnet can attract small pieces of magnetic substances like iron, steel, cobalt, nickel, etc. The attraction is maximum at poles.
- Like poles repel each other and unlike poles attract each other.

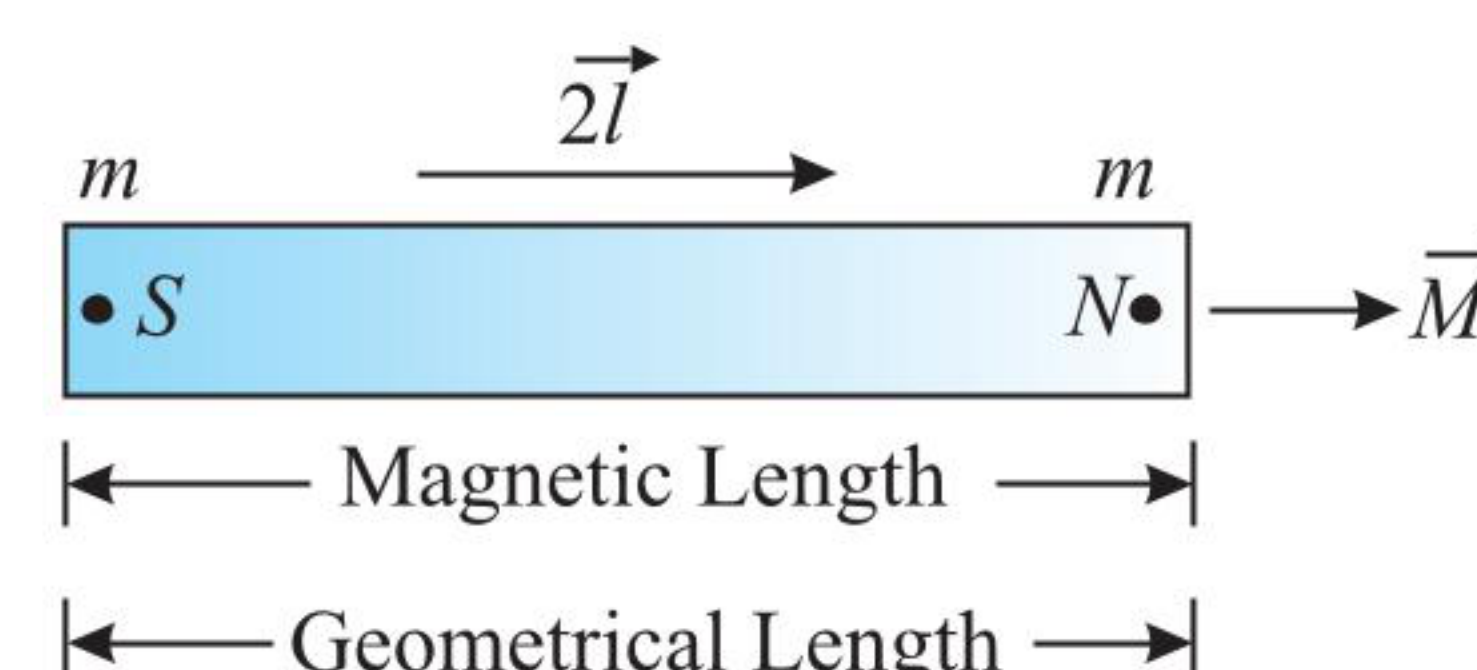
Confirm test of polarity (magnetism) is repulsion and not attraction. Attraction can also occur between a pole of a magnet and a piece of unmagnetized material.

- Breaking a permanent magnet in half does not yield one north pole and one south pole. Instead, two new magnets, each with its own north and south pole result. Unlike electric charge, which exists as separate positive (proton) and negative (electron) charges, no separate magnetic monopoles (isolated north and south poles) exist. Scientists have carried out extensive searches for magnetic monopoles, but nothing has been found. The discussion of the source of magnetism in this chapter will help you understand why there are no magnetic monopoles.



- The poles are situated near the ends of the magnet and not exactly at the ends. The distance between the two poles of a bar magnet is called the magnetic length of the magnet.

Magnetic length is a vector from S-pole of the magnet to its N-pole and is denoted by $2\vec{l}$ [figure].



Since the magnetic poles are situated within the magnet, the distance between its two magnetic poles called its magnetic length is always less than its geometric length.

A line joining north and south poles of a freely suspended magnet is called **magnetic axis**. A vertical plane passing through

the magnetic axis (i.e., S-N of a freely suspended magnet) is called the **magnetic meridian**.

STRENGTH OF BAR MAGNET

The strength of a bar magnet is characterized by its magnetic dipole moment. If the pole strength of either pole of a bar magnet is m and the separation between its poles is $2l$, then the magnetic dipole moment of a bar magnet is given as

$$\mu = m(2l) \quad \dots(i)$$

The symbol used for magnetic dipole moment is either μ or M . Like electric dipole moment, magnetic dipole moment is also a vector quantity with direction taken from south-pole to north-pole. The unit used for measurement of magnetic dipole moment is 'ampere-meter square' or 'A-m²'. Magnetic dipole moment of a bar magnet in general also called as 'magnetic moment' of the magnet.

ILLUSTRATION 3.1

A bar magnet of magnetic moment 5.0 Am^2 has poles 20 cm apart. Calculate the pole strength.

Sol. The magnetic moment $M = 5.0 \text{ Am}^2$

The separation between its poles

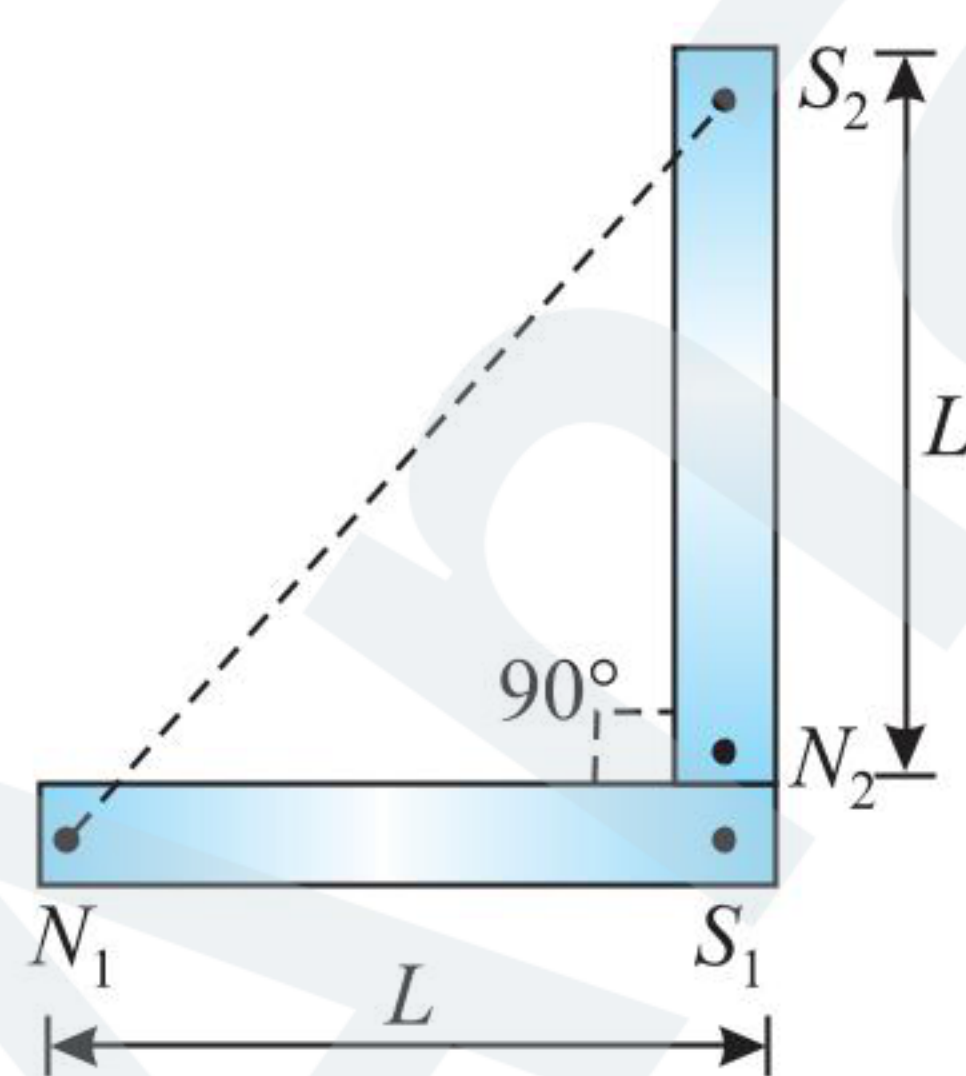
$$2l = 20 \text{ cm} = 20 \times 10^{-2} \text{ m}$$

$$\text{Pole strength, } m = \frac{M}{2l} = \frac{5.0 \text{ Am}^2}{20 \times 10^{-2} \text{ m}} = 25 \text{ Am}$$

ILLUSTRATION 3.2

Two identical thin bar magnets, each of length L and pole-strength m are placed at right angles to each other, with the N-pole of one touching the south pole of the other. Find the magnetic moment of the system.

Sol. When the magnets N_1S_1 and N_2S_2 each of pole strength m and length L are placed at right angles to each other (figure), the system behaves as a magnet, whose pole strength is m and length is N_1S_2 .



$$\text{Now, } N_1S_2 = \sqrt{N_1S_1^2 + N_2S_2^2} = \sqrt{L^2 + L^2} = \sqrt{2} L$$

Therefore, magnetic moment of the system,

$$M = \text{Pole strength} \times N_1S_2 = m \times \sqrt{2} L = \sqrt{2} mL$$

MAGNETIC FIELD

Magnetic Field is the space around a magnet (or a conductor carrying current) in which its magnetic effect can be experienced. The SI unit of strength of magnetic field (also known as magnetic

induction or magnetic induction field or magnetic flux density) is tesla (T). It is measured in terms of force on a charge moving inside the magnetic field or by using the concept of magnetic flux linked with a surface.

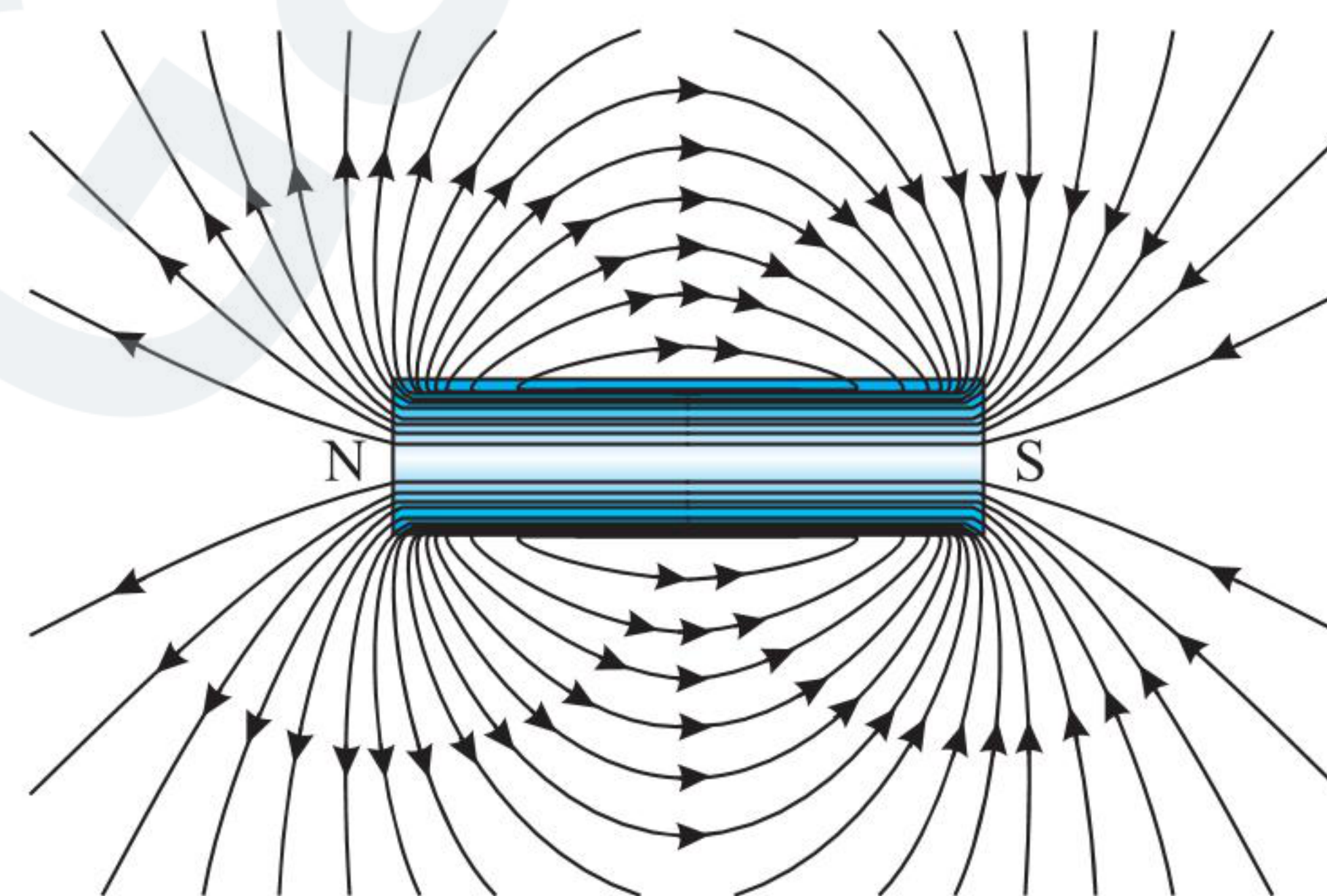
$$1 \text{ tesla (T)} = 1 \text{ newton ampere}^{-1} \text{metre}^{-1} (\text{N A}^{-1} \text{ m}^{-1}) \\ = 1 \text{ weber metre}^{-2} (\text{Wb m}^{-2})$$

Note:

1. Strength of magnetic field is measured in gauss (G).
1 gauss (G) = 10^{-4} tesla (T)
2. The strength of magnetic field B is also known as magnetic field.

MAGNETIC FIELD LINES

Like an electric field, a magnetic field is represented using field lines. The magnetic field vector is always tangent to the **magnetic field lines**. The magnetic field lines from a permanent bar magnet are shown in figure.



- (i) A magnetic field line is an imaginary curve, the tangent to which at any point P gives the direction of $\vec{B}$ at that point.
- (ii) The magnetic field lines are closed continuous loops extending through the body of the magnet.
- (iii) The number of field lines drawn per unit cross-sectional area is proportional to B . If the field is uniform, the field lines are evenly spaced.
- (iv) Magnetic field lines never intersect each other.
- (v) Externally, magnetic field lines appear to originate on north poles and terminate on south poles, but these field lines are actually closed loops that penetrate the magnet itself. This formation of loops is an important difference between electric and magnetic field lines

The direction of the magnetic field is established in terms of the direction in which a compass needle points. A compass needle, with a north pole and a south pole, will orient itself so that its north pole points in the direction of the magnetic field.

COULOMB'S LAW OF FORCE BETWEEN TWO MAGNETIC POLES

Isolated magnetic poles do not exist. But the idea of a magnetic pole is useful in certain analogies. We will discuss later how the idea of a magnetic dipole is a more acceptable concept.

The force of attraction or repulsion between two magnetic poles of strengths m_1 and m_2 placed at a distance r apart is given by

$$F \propto \frac{m_1 m_2}{r^2} \quad \text{or} \quad F = k \frac{m_1 m_2}{r^2} \quad \dots(i)$$

where k is a constant of proportionality whose value depends on (i) the system of units selected and (ii) the medium in which the poles are placed.

$$\text{In SI units: } k = \frac{\mu_0}{4\pi} = 10^{-7} \text{ Wb/Am}$$

Equation (i) is of limited application as it applies to long, thin magnets with well separated poles.

ILLUSTRATION 3.3

Two equal magnetic poles placed 5 cm in air attract each other with a force of 14.4×10^{-4} N. How far from each other should they be placed so that the force of attraction will be 1.6×10^{-4} N?

Sol. Let m be the pole strength of each pole.

In first case: $F = 14.4 \times 10^{-4}$ N; $r = 5 \text{ cm} = 0.05 \text{ m}$

$$\text{Now, } F = \frac{\mu_0}{4\pi} \cdot \frac{m \times m}{r^2}$$

$$\text{or } 14.4 \times 10^{-4} = 10^{-7} \times \frac{m^2}{(0.05)^2}$$

$$\text{or } m = \pm 6 \text{ Am}$$

In second case: Let r' be the distance between the poles, when force of attraction becomes $F' = 1.6 \times 10^{-4}$ N.

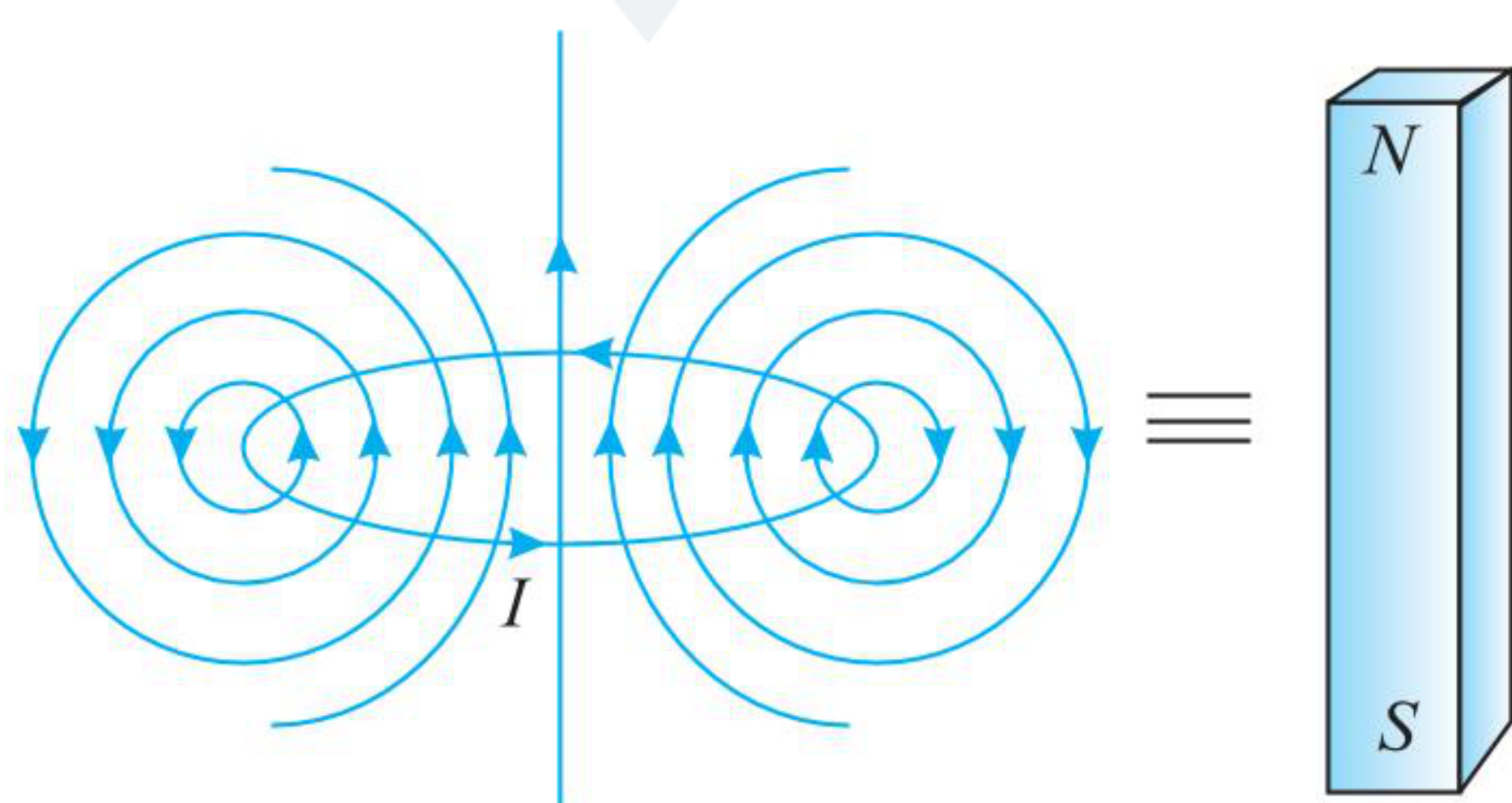
$$\text{Now, } F' = \frac{\mu_0}{4\pi} \cdot \frac{m \times m}{r'^2}$$

$$\text{or } 1.6 \times 10^{-4} = 10^{-7} \times \frac{6 \times 6}{r'^2}$$

$$\text{or } r' = 0.15 \text{ m}$$

A SMALL CURRENT-CARRYING COIL AS A MAGNETIC DIPOLE

Consider a circular coil-carrying current I . Suppose that the current flows in anticlockwise direction, when seen from above (figure). The magnetic field lines due to each elementary portion of the circular coil will be of the shape of circular loops near that portion and almost straight near the centre of the circular coil. From right hand thumb rule, it follows that the magnetic field lines seem to enter at the lower face of the coil and leave at its upper face. Thus, the lower face of the coil, through which magnetic field lines enter, behaves as the south pole; while the upper face, through which the field lines leave, behaves as the north pole. Hence, the current carrying circular coil behaves as a magnetic dipole.



The figure below shows the magnetic field produced by a bar magnet and that of a current carrying coil which looks similar except the length of bar magnet but if we look at the figure which shows the magnetic field in surrounding of a small magnetic dipole and that of a small current-carrying coil. Both the configurations of magnetic field lines look almost identical. Thus a very small current-carrying coil can be regarded as a magnetic dipole and all the results we have obtained for a magnetic dipole can be used for such small current-carrying loops with magnetic dipole moment of dipole replaced with the magnetic moment of the coil.

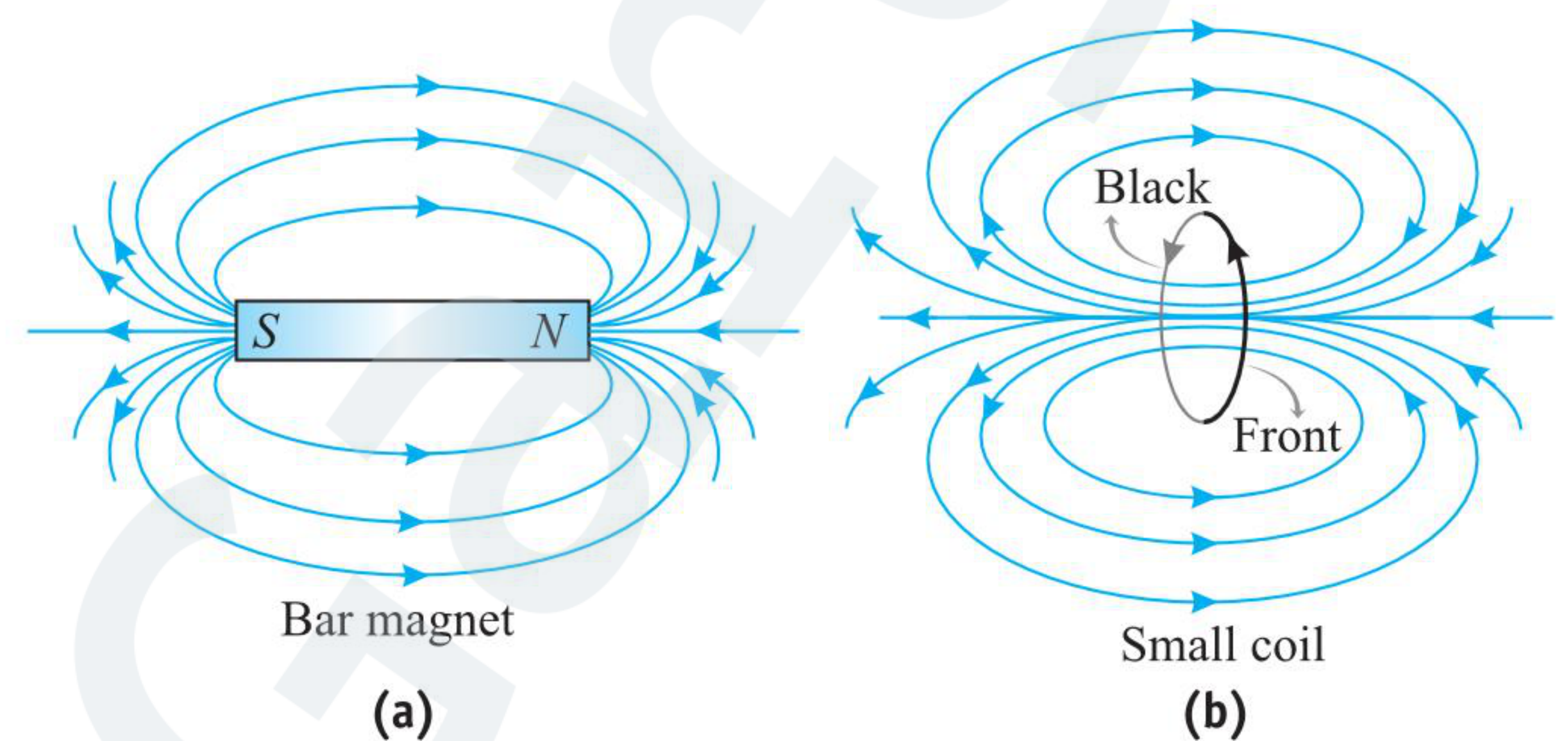
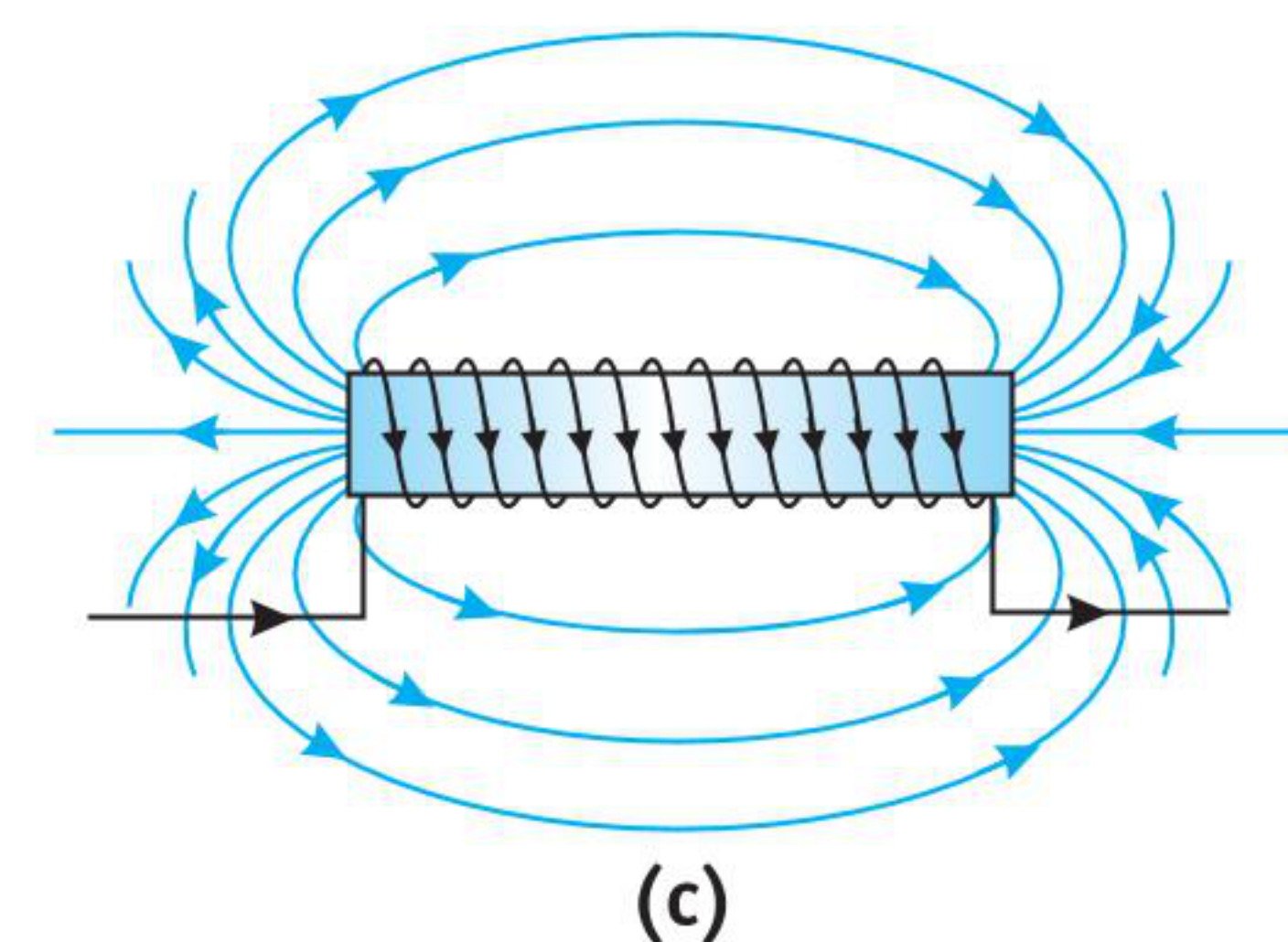


Figure (c) shows a tightly wound long solenoid and the magnetic induction produced by it by the magnetic field lines configuration. This magnetic field lines configuration is almost same as that produced by a bar magnet. Thus a current-carrying solenoid can be considered to be behaving like a bar magnet with magnetic moment given as

$$M = NiA$$

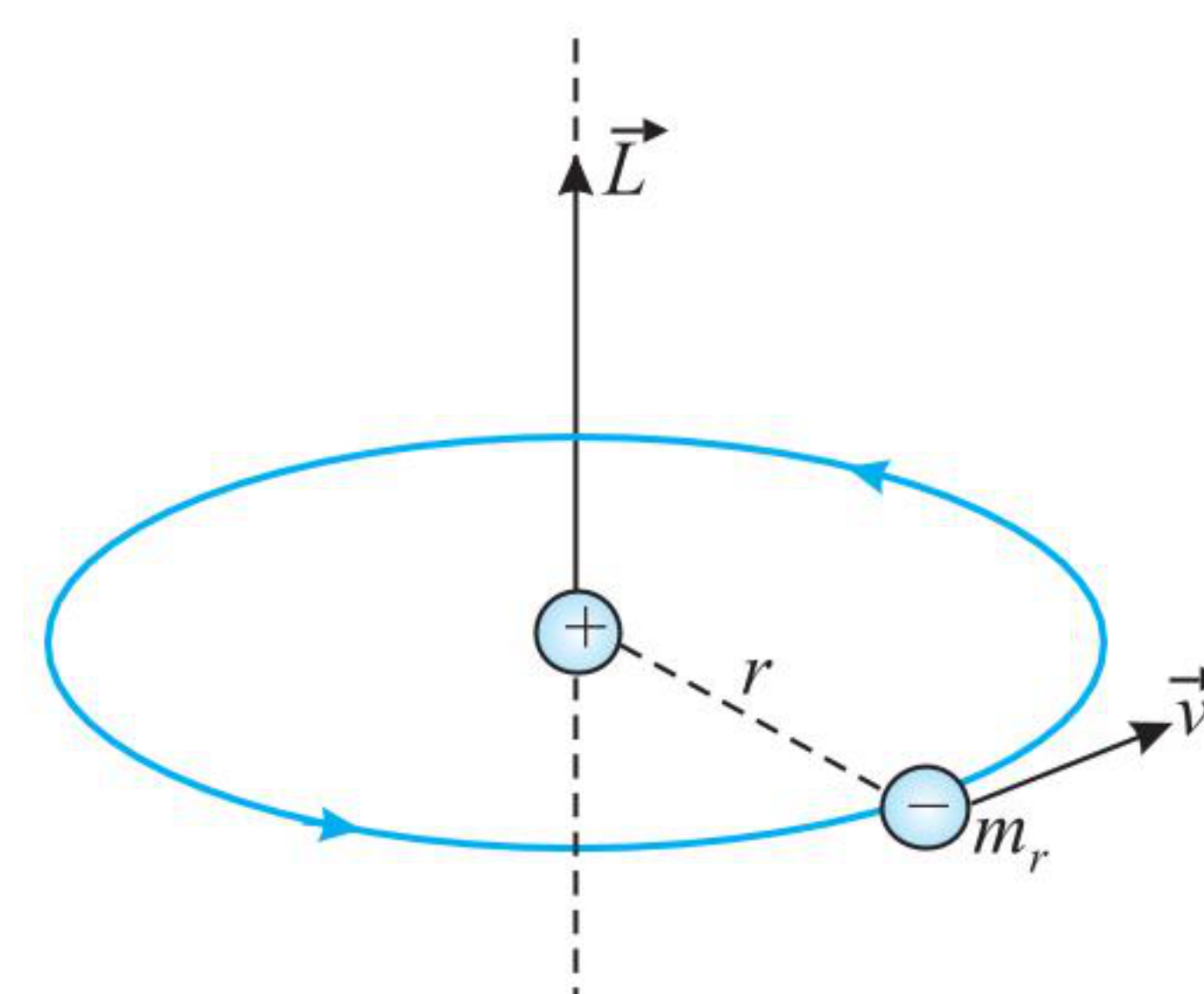


ATOM AS A MAGNETIC DIPOLE

In an atom, electrons revolve around the nucleus and as such the circular orbits of electrons may be considered as the small current loops. Due to this, an atom possesses magnetic dipole moment and hence behaves as a magnetic dipole.

Expression for magnetic moment: Suppose that an electron of mass m_e and charge e revolves in a circular orbit of radius r around the positive nucleus in anticlockwise direction, as seen in figure. The angular momentum of the electron due to its orbital motion is given by

$$L = m_e v r \quad \dots(i)$$



For the sense of orbital motion of electron shown in the figure, the angular momentum vector $\vec{L}$ acts along normal to the plane of the electron orbit and in upward direction.

Suppose that the period of orbital motion of the electron is T . Then, the electron crosses any point on its orbit after every T seconds or $1/T$ times in one second.

Therefore, orbital motion of electron is equivalent to a current,

$$I = e \left(\frac{1}{T} \right)$$

Now, the period of revolution of the electron is given by

$$T = \frac{2\pi r}{v}$$

$$\text{Therefore, } I = e \left(\frac{1}{2\pi r/v} \right) = \frac{ev}{2\pi r}$$

The area of the electron orbit (area of current loop), $A = \pi r^2$
Therefore, the magnetic dipole moment of the atom,

$$M = \frac{ev}{2\pi r} \times \pi r^2 = \frac{evr}{2}$$

For rotating body, the ratio of magnetic moment and angular momentum is a constant, which is equal to $\frac{q}{2m}$, where q is the charge and m is the mass of the body. Here in this case, electron is rotating around nucleus. Hence here we can write

$$M = \left(\frac{e}{2m_e} \right) L \quad \dots(\text{ii})$$

In vector notation,

$$\vec{M} = - \left(\frac{e}{2m_e} \right) \vec{L} \quad \dots(\text{iii})$$

The negative sign has been included for the reason that electron has negative charge. Equation (iii) tells that the magnetic dipole moment vector is directed in a direction opposite to that of angular momentum vector.

According to Bohr's theory, angular momentum of electron in a stationary orbit can have only those values, which are integral multiples of $\frac{h}{2\pi}$ i.e.

$$L = n \frac{h}{2\pi}, \text{ where } n = 1, 2, 3 \quad \dots(\text{iv})$$

From Eqs. (ii) and (iv), we have

$$M = \left(\frac{e}{2m_e} \right) n \frac{h}{2\pi}$$

$$\text{or } M = n \left(\frac{eh}{4\pi m_e} \right) \quad \dots(\text{v})$$

Equation (v) gives the magnetic moment of an electron due to its orbital motion.

Bohr magneton: It is defined as the magnetic dipole moment associated with an atom due to orbital motion of an electron in the first orbit of hydrogen atom.

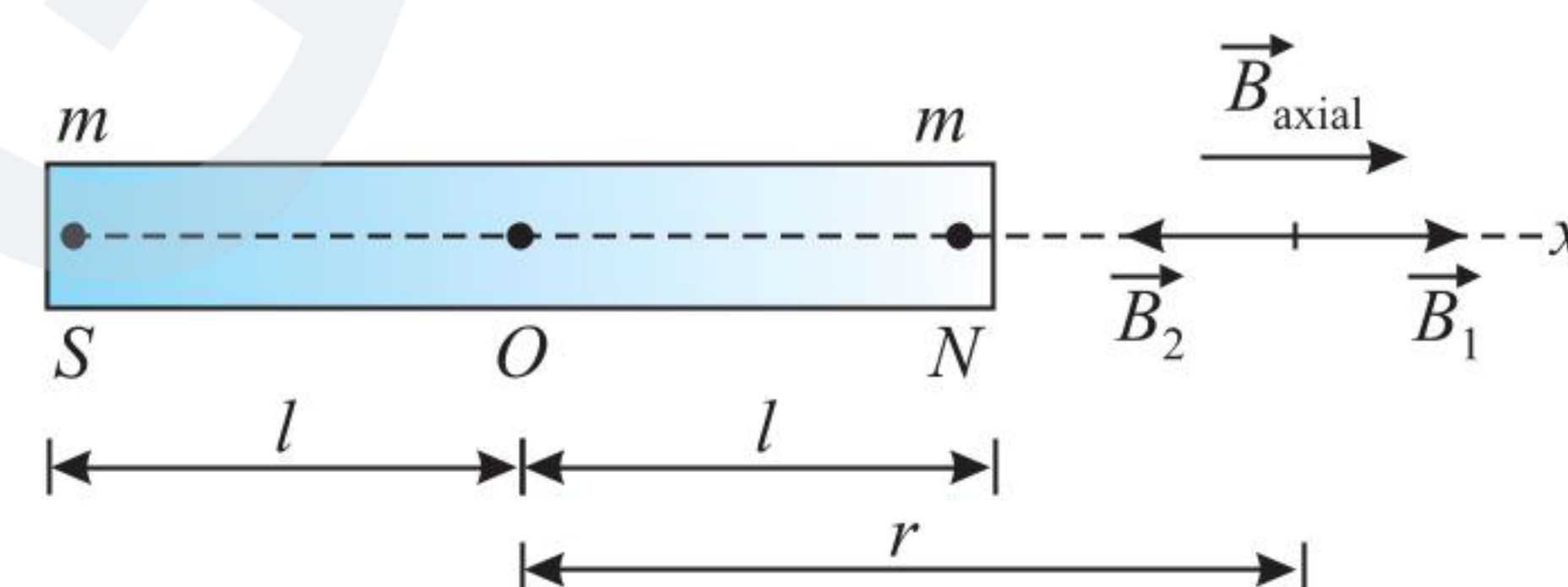
From Eq. (v), it follows that $\frac{eh}{4\pi m_e}$ is the least value of the magnetic moment. This natural unit of magnetic moment is called Bohr magneton. It is denoted by μ_B . Its value is

$$\mu_B = \frac{(1.6 \times 10^{-19} \text{ C}) \times (6.62 \times 10^{-34} \text{ Js})}{4\pi \times (9.1 \times 10^{-31} \text{ kg})} = 9.27 \times 10^{-24} \text{ Am}^2$$

We know that in an atom, in addition to orbital motion, an electron has got spin motion also. The electron possesses magnetic moment due to its spin motion also. The total magnetic moment of the electron is the vector sum of its magnetic moments due to orbital and spin motion.

MAGNETIC FIELD ON AXIAL LINE OF A BAR MAGNET

Consider a bar magnet NS, with pole strength m and length $2l$. P is a point located at a distance r from the center and on the axis of magnet. The two magnetic poles produce magnetic induction at point P in opposite directions as shown in figure.



The net magnetic field $\vec{B}_{\text{axial}}$ at point P is given as

$$\vec{B}_{\text{axial}} = \vec{B}_1 + \vec{B}_2$$

$$\text{Now, } |\vec{B}_1| = \frac{\mu_0}{4\pi} \cdot \frac{m}{NP^2} = \frac{\mu_0}{4\pi} \cdot \frac{m}{(r-l)^2} \text{ (along PX)}$$

$$\text{and } |\vec{B}_2| = \frac{\mu_0}{4\pi} \cdot \frac{m}{SP^2} = \frac{\mu_0}{4\pi} \cdot \frac{m}{(r+l)^2} \text{ (along PS)}$$

It follows that $|\vec{B}_1|$ is greater than $|\vec{B}_2|$.

Since $\vec{B}_1$ and $\vec{B}_2$ act along the same line but in opposite directions,

$$\therefore |\vec{B}_{\text{axial}}| = |\vec{B}_1| - |\vec{B}_2| \quad \text{(along PX)}$$

$$\begin{aligned} \text{or } B_{\text{axial}} &= \frac{\mu_0}{4\pi} \cdot \frac{m}{(r-l)^2} - \frac{\mu_0}{4\pi} \cdot \frac{m}{(r+l)^2} \\ &= \frac{\mu_0}{4\pi} \cdot m \left[\frac{(r+l)^2 - (r-l)^2}{(r^2 - l^2)^2} \right] = \frac{\mu_0}{4\pi} \cdot \frac{m(4rl)}{(r^2 - l^2)^2} \end{aligned}$$

Now, $m(2l) = M$, magnitude of the magnetic dipole moment of the magnet.

$$\therefore B_{\text{axial}} = \frac{\mu_0}{4\pi} \cdot \frac{2Mr}{(r^2 - l^2)^2} \quad \text{(along PX)} \quad \dots(\text{i})$$

It may be noted that the direction of magnetic field due to a bar magnet at a point on its axial line is from S-pole to N-pole i.e. same as that of magnetic dipole moment of the bar magnet. Therefore, Eq. (i) may be expressed as

$$\vec{B}_{\text{axial}} = \frac{\mu_0}{4\pi} \cdot \frac{2\vec{M}r}{(r^2 - l^2)^2} \quad \dots(\text{ii})$$

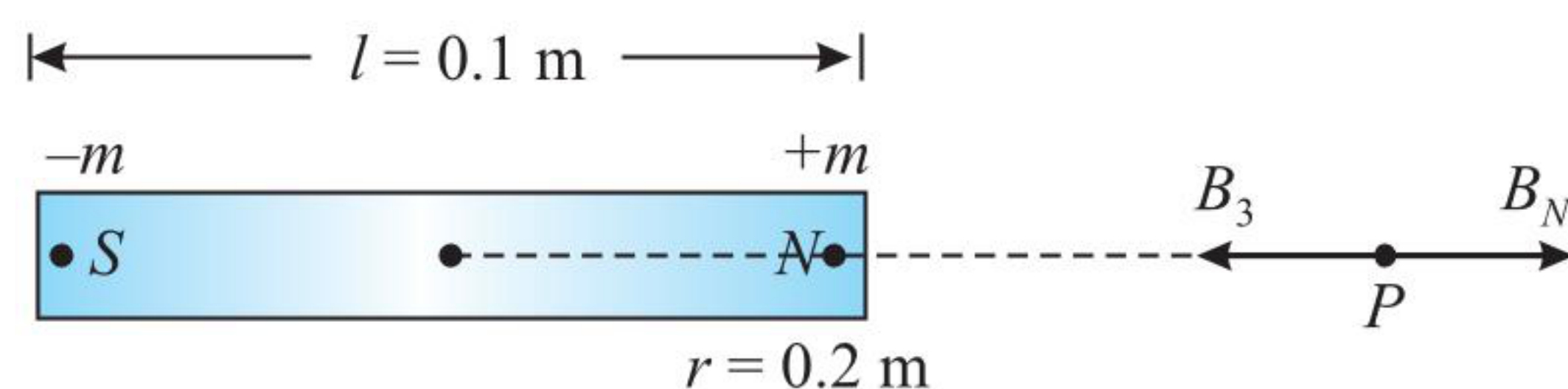
If bar magnet is of very small length. For a bar magnet of very small length, $l \ll r$, and hence in Eq. (ii), l^2 can be neglected in comparison to r^2 . Therefore, for a very short bar magnet,

$$B_{\text{axial}} = \frac{\mu_0}{4\pi} \cdot \frac{2M}{r^3} \quad (\text{along } PX) \quad \dots(\text{iii})$$

ILLUSTRATION 3.4

A bar magnet is 0.1 m long and its pole strength is 12 Am. Find the magnetic induction at a point on its axis at a distance of 0.2 m from its centre.

Sol. Net magnetic induction due to bar magnet at point P is given by the vector sum of the magnetic inductions at P due to the two poles is given as



$$B_P = B_N - B_S$$

$$\Rightarrow B_P = \frac{\mu_0}{4\pi} \cdot \frac{m}{(r-l/2)^2} - \frac{\mu_0}{4\pi} \cdot \frac{m}{(r+l/2)^2}$$

$$\Rightarrow B_P = \frac{\mu_0}{4\pi} \left[\frac{2rl}{(r^2 - l^2/4)^2} \right] = \frac{32\mu_0 mrl}{4\pi(4r^2 - l^2)^2}$$

$$\Rightarrow B_P = \frac{32 \times 10^{-7} \times 12 \times 0.2}{(4(0.2)^2 - (0.1)^2)^2}$$

$$\Rightarrow B_P = 3.4 \times 10^{-4} \text{ T}$$

MAGNETIC FIELD ON EQUATORIAL LINE OF A BAR MAGNET

Consider a bar magnet NS, whose each pole is of strength m . Let $2l$ be the magnetic length of the bar magnet and O be its centre. Let P be a point on the equatorial line of the magnet at a distance r from the centre of the magnet.

Let $\vec{B}_1$ and $\vec{B}_2$ be the magnetic fields at point P due to N-pole and S-pole of the bar magnet. Then, resultant magnetic field at point P due to the bar magnet is given by

$$\vec{B}_{\text{equi}} = \vec{B}_1 + \vec{B}_2$$

$$\text{Now, } |\vec{B}_1| = |\vec{B}_2| = \frac{\mu_0}{4\pi} \cdot \frac{m}{(r^2 + l^2)} \quad (\text{along } NP)$$

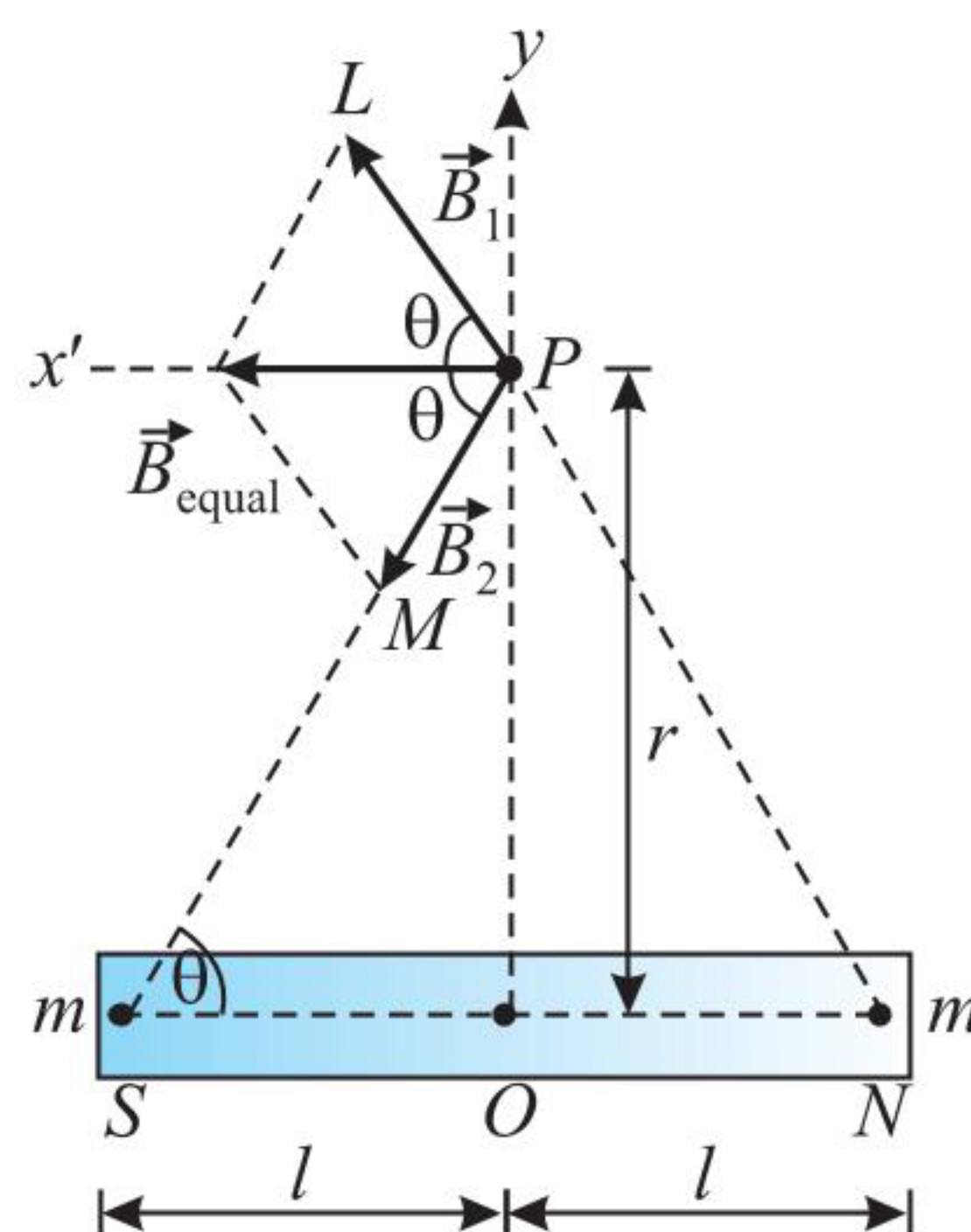
$$\text{and } |\vec{B}_2| = \frac{\mu_0}{4\pi} \cdot \frac{m}{SP^2} = \frac{\mu_0}{4\pi} \cdot \frac{m}{(r^2 + l^2)} \quad (\text{along } PS)$$

The two magnetic poles produce magnetic field of equal magnitude at point P due to symmetry along the directions as shown in figure.

Therefore, resultant magnetic field at point P due to the bar magnet,

$$|\vec{B}_{\text{equi}}| = 2|\vec{B}_1|\cos\theta$$

$$\text{As, } |\vec{B}_1| = |\vec{B}_2| = \frac{\mu_0}{4\pi} \cdot \frac{m}{(r^2 + l^2)}$$



$$\text{or } B_{\text{equi}} = \frac{\mu_0}{4\pi} \cdot \frac{2m}{(r^2 + l^2)} \cos\theta$$

From right-angled $\triangle SOP$,

$$\cos\theta = \frac{SO}{SP} = \frac{l}{(r^2 + l^2)^{1/2}}$$

$$\text{Hence, } B_{\text{equi}} = \frac{\mu_0}{4\pi} \cdot \frac{2m}{(r^2 + l^2)} \cdot \frac{l}{(r^2 + l^2)^{1/2}}$$

$$\text{or } B_{\text{equi}} = \frac{\mu_0}{4\pi} \cdot \frac{M}{(r^2 + l^2)^{3/2}} \quad (\text{along } PX') \quad \dots(\text{iv})$$

Here, $M = 2ml$ is magnetic dipole moment of the bar magnet.

It may be noted that direction of magnetic field due to a bar magnet on its equatorial line is from N-pole to S-pole of the magnet i.e. in a direction opposite to that of the magnetic dipole moment of the magnet. Therefore, Eq. (iv) may be expressed as

$$\vec{B}_{\text{equi}} = -\frac{\mu_0}{4\pi} \cdot \frac{\vec{M}}{(r^2 + l^2)^{3/2}} \quad \dots(\text{v})$$

For a bar magnet of very small length, $l \ll r$ and hence in Eq. (v), l^2 can be neglected in comparison to r^2 . Therefore, for a very short bar magnet,

$$B_{\text{equi}} = \frac{\mu_0}{4\pi} \cdot \frac{M}{r^3} \quad (\text{along } PX') \quad \dots(\text{vi})$$

Important Points:

From Eqs. (iii) and (vi), it follows that magnetic field at a point at a certain distance on the equatorial line of a short bar magnet is just one-half of that at the same distance on its axial line.

Axial position

$$\vec{B}_{\text{axial}} = \frac{\mu_0}{4\pi} \frac{2M}{r^3} = K \frac{2M}{r^3}$$

Equatorial position

$$B_{\text{equi}} = \frac{\mu_0}{4\pi} \frac{M}{r^3} = K \frac{M}{r^3}$$

ILLUSTRATION 3.5

A bar magnet of length 0.1 m has a pole strength of 50 A m. Calculate the magnetic field at distance of 0.2 m from its center on (a) its axial line and (b) its equatorial line.

Sol. Here, $m = 50 \text{ Am}$; $r = 0.2 \text{ m}$; $2l = 0.1 \text{ m}$ or $l = 0.05 \text{ m}$

Therefore, $M = m(2l) = 50 \times 0.1 = 5 \text{ Am}^2$

$$\text{(a) } B_{\text{axial}} = \frac{\mu_0}{4\pi} \cdot \frac{2Mr}{(r^2 - l^2)^2}$$

$$= \frac{10^{-7} \times 2 \times 5 \times 0.2}{(0.2^2 - 0.05^2)^2} = \frac{10^{-7} \times 2 \times 5 \times 0.2}{(0.0375)^2}$$

$$= 1.42 \times 10^{-4} \text{ T}$$

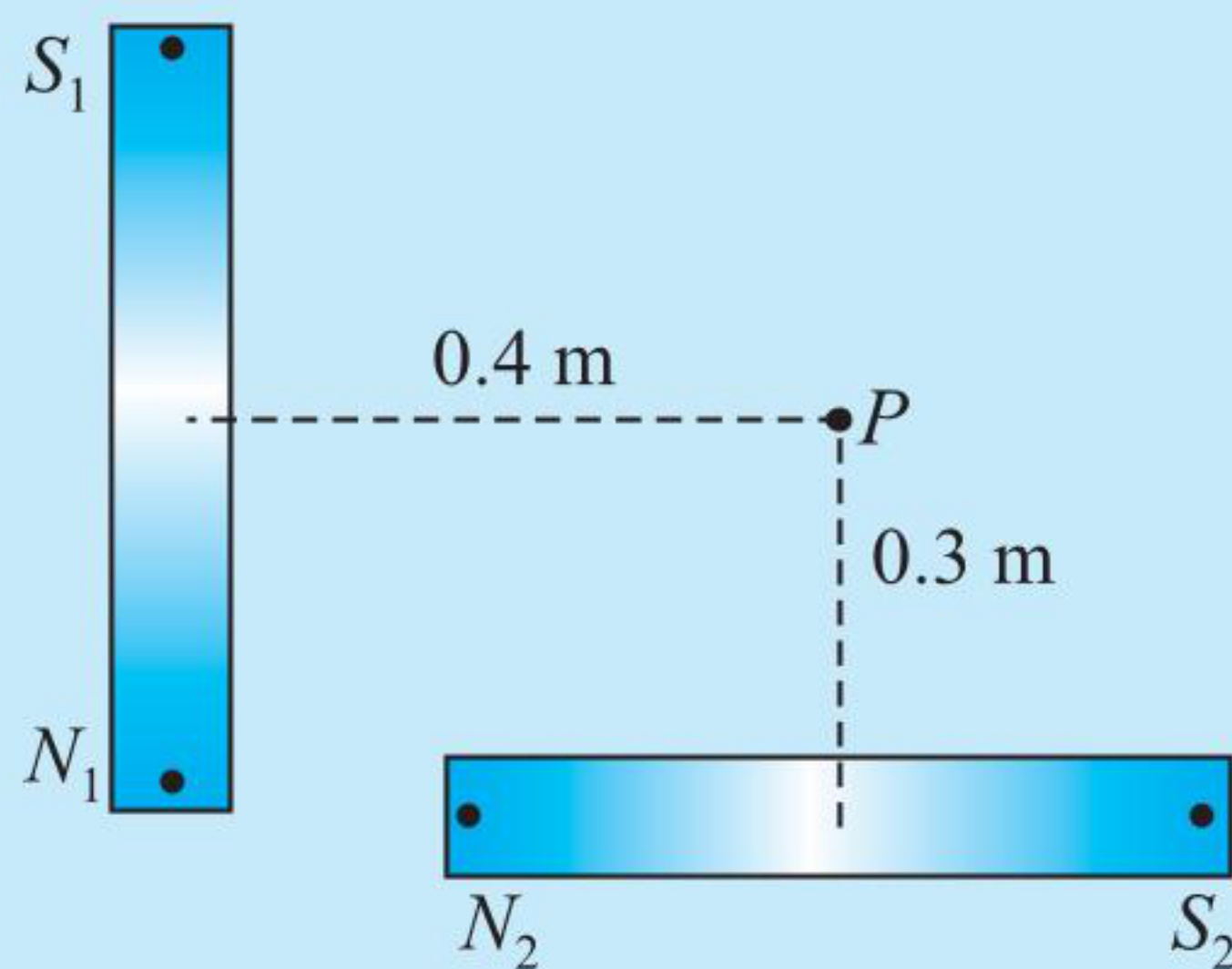
$$(b) B_{\text{equi}} = \frac{\mu_0}{4\pi} \cdot \frac{M}{(r^2 + l^2)^{3/2}}$$

$$= \frac{10^{-7} \times 5}{(0.2^2 + 0.05^2)^{3/2}} = \frac{10^{-7} \times 5}{(0.0425)^{3/2}} = \frac{5 \times 10^{-7}}{0.00876}$$

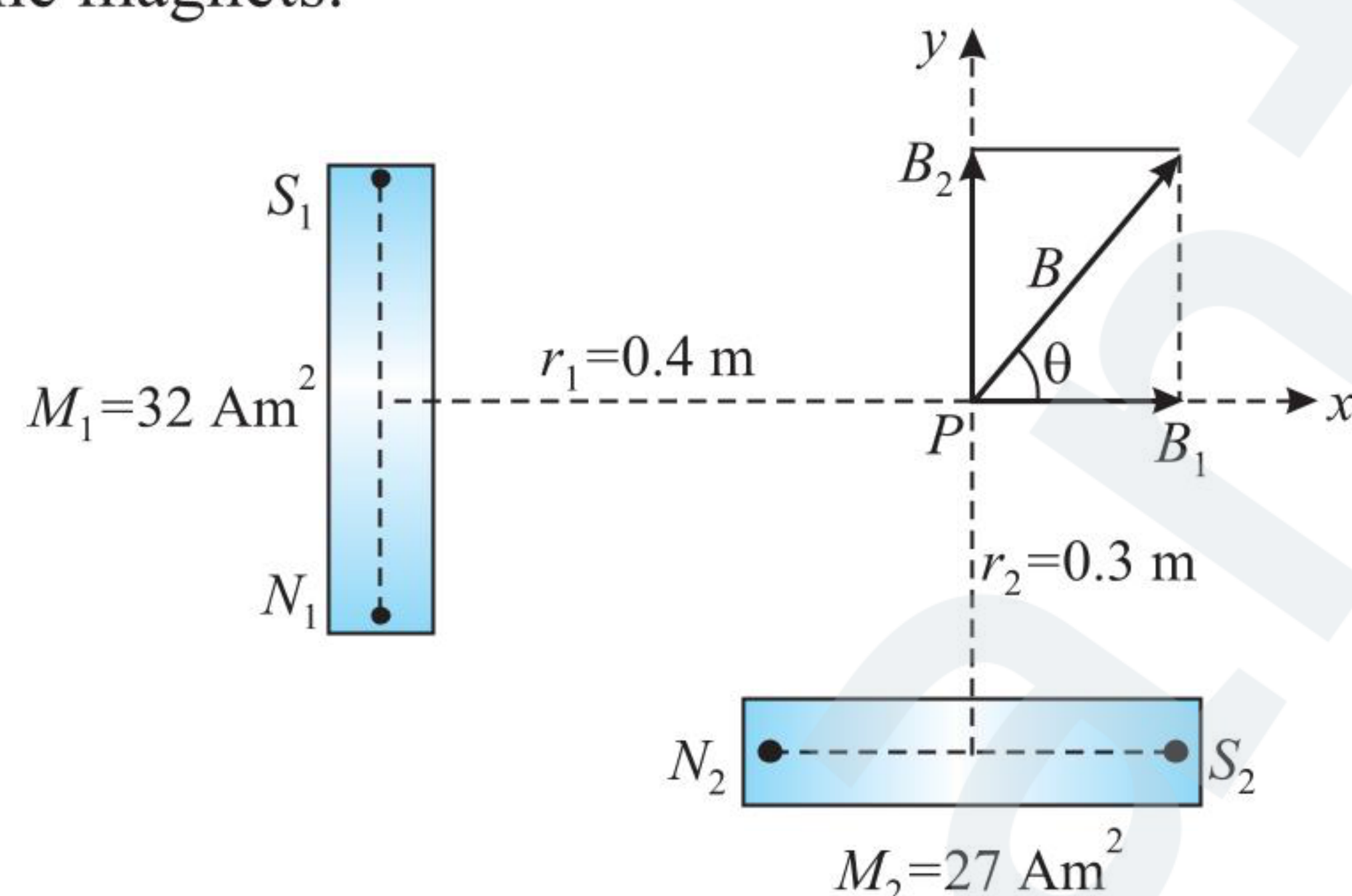
$$= 5.708 \times 10^{-5} \text{ T}$$

ILLUSTRATION 3.6

Two short bar magnets N_1S_1 and N_2S_2 of magnetic moments 32 Am^2 and 27 Am^2 are placed on the table as shown in figure. Find the magnitude and direction of the magnetic fields produced by these magnets at point P located on equatorial lines of both the magnets at distances of 0.4 m and 0.3 m respectively from their centers.



Sol. As shown in figure, the point P lies on the equatorial line of both the magnets.



Let B_1 and B_2 be magnetic fields at point P due to the magnets N_1S_1 and N_2S_2 respectively. Then,

$$B_1 = \frac{\mu_0}{4\pi} \cdot \frac{M_1}{r_1^3} = \frac{10^{-7} \times 32}{(0.4)^3} = 0.5 \times 10^{-4} \text{ T} \quad (\text{along PY})$$

$$\text{and } B_2 = \frac{\mu_0}{4\pi} \cdot \frac{M_2}{r_2^3} = \frac{10^{-7} \times 27}{(0.3)^3} = 10^{-4} \text{ T} \quad (\text{along PX})$$

Therefore, resultant magnetic field at point P ,

$$B = \sqrt{B_1^2 + B_2^2} = \sqrt{(0.5 \times 10^{-4})^2 + (10^{-4})^2} = 1.12 \times 10^{-4} \text{ T}$$

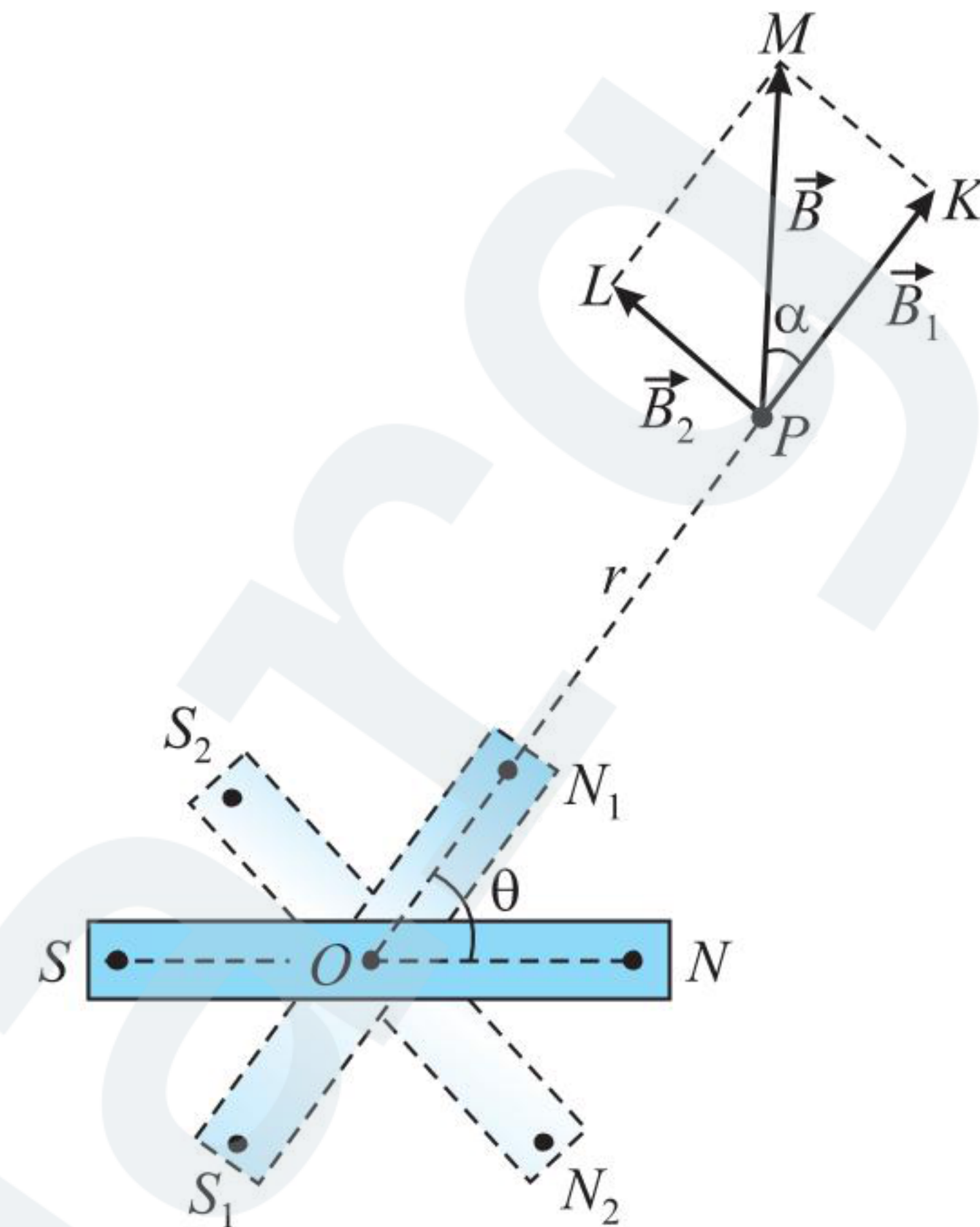
If the direction of B makes an angle θ with the X -axis (or N_1S_1), then

$$\tan \theta = \frac{B_2}{B_1} = \frac{10^{-4}}{0.5 \times 10^{-4}} = 2$$

$$\Rightarrow \theta = \tan^{-1} 2$$

MAGNETIC FIELD AT ANY POINT DUE TO A SHORT MAGNETIC DIPOLE

In the given figure, a point P is at a distance r from a magnetic dipole NS of dipole moment M . The line joining point P to the center of dipole is making an angle θ with the axis of dipole.



The dipole moment $\vec{M}$ of the dipole (NS) can be resolved into two components:

- The component $M \cos \theta$ along OP and
- The component $M \sin \theta$ along a direction perpendicular to OP .

Here we can consider the magnetic dipole NS to be equivalent to the combination of two magnetic dipoles i.e. one dipole N_1S_1 having dipole moment $M \cos \theta$ and another magnetic dipole N_2S_2 (placed perpendicular to N_1S_1) having magnetic dipole moment $M \sin \theta$.

The point P lies on the axial line of dipole N_1S_1 . Therefore, magnetic field intensity at point P due to the dipole N_1S_1 ,

$$|\vec{B}_1| = \frac{\mu_0}{4\pi} \cdot \frac{2M \cos \theta}{r^3} \quad (\text{along PK})$$

Further, the point P lies on the equatorial line of dipole N_2S_2 . Therefore, magnetic field intensity at point P due to the dipole N_2S_2 ,

$$|\vec{B}_2| = \frac{\mu_0}{4\pi} \cdot \frac{M \sin \theta}{r^3} \quad (\text{along PL, perpendicular to PK})$$

The resultant magnetic field intensity at point P ,

$$\vec{B} = \vec{B}_1 + \vec{B}_2$$

$$\text{or } B = \sqrt{|\vec{B}_1|^2 + |\vec{B}_2|^2}$$

$$= \sqrt{\left(\frac{\mu_0}{4\pi} \cdot \frac{2M \cos \theta}{r^3} \right)^2 + \left(\frac{\mu_0}{4\pi} \cdot \frac{M \sin \theta}{r^3} \right)^2}$$

$$= \frac{\mu_0}{4\pi} \cdot \frac{M}{r^3} \sqrt{4 \cos^2 \theta + \sin^2 \theta}$$

$$\text{or } B = \frac{\mu_0}{4\pi} \cdot \frac{M \sqrt{3 \cos^2 \theta + 1}}{r^3} \quad \dots(i)$$

Equation (i) gives the magnitude of magnetic field intensity due to a short magnetic dipole at a distance r from its centre in a direction making an angle θ with the dipole.

To find the direction of magnetic field intensity due to the dipole, suppose that it makes an angle α with the direction of $\vec{B}_1$ i.e. with the line OK . Then, from the right-angled $\triangle PKM$, we have

$$\tan \alpha = \frac{MK}{PK} = \frac{|\vec{B}_2|}{|\vec{B}_1|} = \frac{\mu_0}{4\pi} \cdot \frac{M \sin \theta}{r^3} \bigg/ \frac{\mu_0}{4\pi} \cdot \frac{2M \cos \theta}{r^3}$$

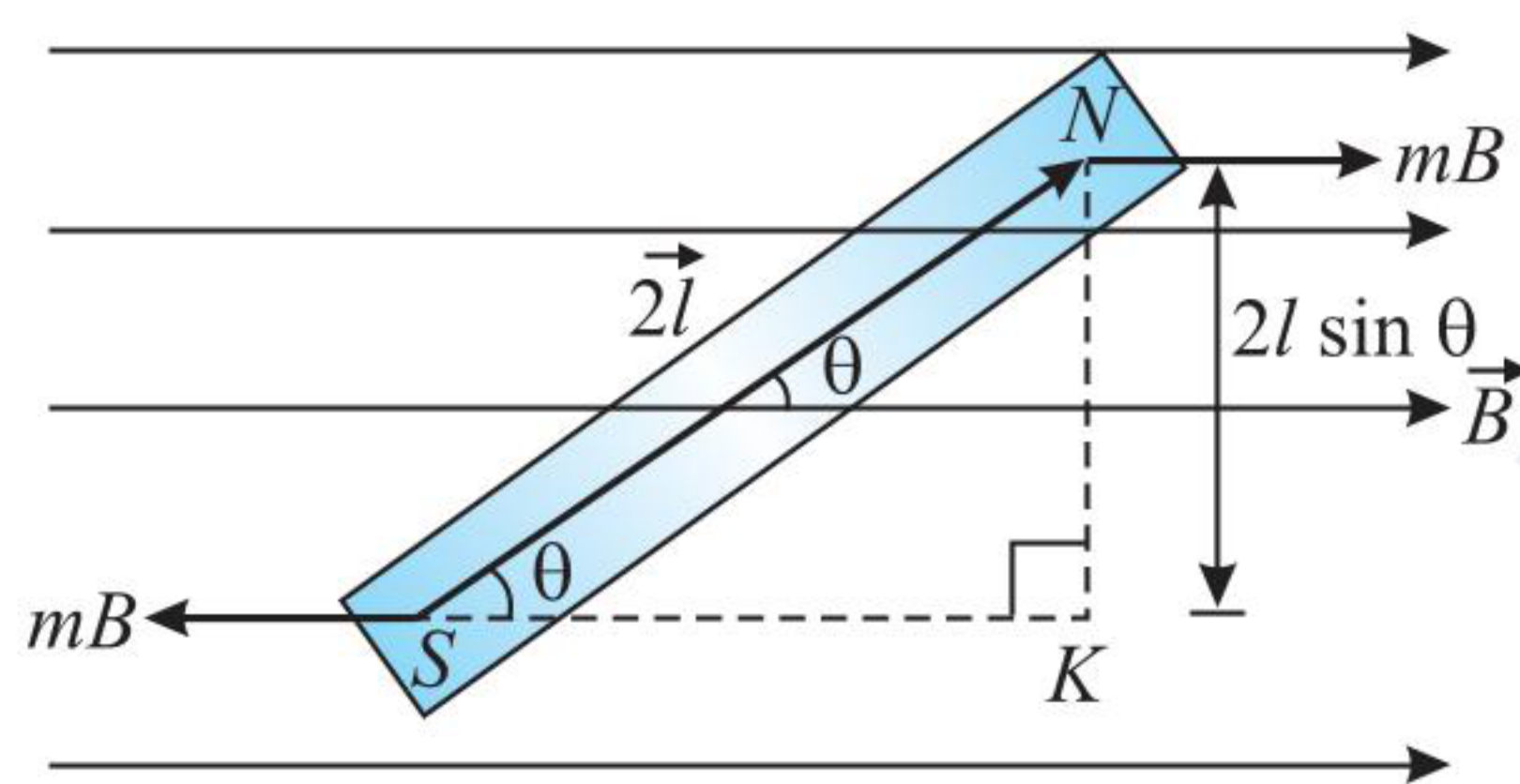
$$\text{or } \tan \alpha = \frac{1}{2} \tan \theta \quad \dots(ii)$$

Equation (ii) can be used to find the value of the angle α and hence the direction of magnetic field intensity due to the short magnetic dipole.

TORQUE ON A BAR MAGNET IN A MAGNETIC FIELD

The figure shows a magnetic dipole of dipole moment M placed in a uniform magnetic induction B . As both of its poles will experience equal and opposite forces as shown in figure, the net magnetic force on the dipole will be zero similar to an electric dipole placed in uniform electric field. Due to equal and opposite forces acting at different lines of action, a couple will be produced and the torque on magnetic dipole due to these couple forces can be given by its analogy with the case of electric dipole already studied in the chapter of electrostatics.

Consider a bar magnet having pole strength m and of length $2l$ be placed in a uniform magnetic field of strength B making an angle θ with the direction of the magnetic field.



Force on N-pole of the magnet = mB (along the direction of magnetic field B)

Force on S-pole of the magnet = mB (opposite to the direction of magnetic field B)

As both of its poles will experience equal and opposite forces as shown in figure, the net magnetic force on the dipole will be zero similar to an electric dipole placed in uniform electric field.

But these two forces constitute a torque and it tends to rotate the magnet in the clockwise direction. The magnitude of the torque is given by

$$\begin{aligned} \tau &= \text{Either force} \times \text{perpendicular distance between the two forces} \\ &= mB \times KN \end{aligned}$$

From the right angled triangle NKS , we have

$$KN = NS \sin \theta = 2l \sin \theta$$

$$\therefore \tau = mB \times 2l \sin \theta = m(2l) B \sin \theta$$

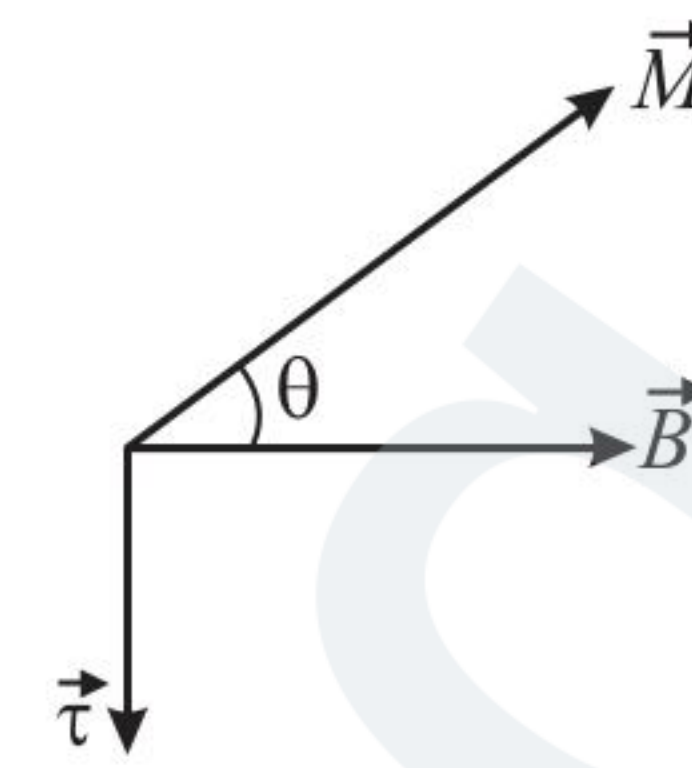
Since $m(2l) = M$, the magnetic dipole moment of the bar magnet, we have

$$\tau = MB \sin \theta \quad \dots(i)$$

In vector notation,

$$\vec{\tau} = \vec{M} \times \vec{B} \quad \dots(ii)$$

If the vectors $\vec{M}$ and $\vec{B}$ are in the plane of paper (figure), then torque vector $\vec{\tau}$, which acts in the direction of $\vec{M} \times \vec{B}$ will be perpendicular to the plane of the paper and in inward direction.



Unit of magnetic dipole moment: From Eq. (i), we have

$$M = \frac{\tau}{B \sin \theta}$$

Therefore, unit of $M = \frac{\text{newton meter}}{\text{tesla}} = \text{NmT}^{-1} = \text{JT}^{-1}$

Therefore, magnetic dipole moment may be measured as newton metre tesla⁻¹ (NmT⁻¹) or as joule tesla⁻¹ (JT⁻¹). It may be measured as ampere metre² (Am²). Since tesla is weber metre⁻², the magnetic dipole moment is dimensionally equivalent to joule metre² weber⁻¹ (J m²Wb⁻¹).

ILLUSTRATION 3.7

A bar magnet has poles of strength 48 Am, which are 25 cm apart. (a) What is the magnetic moment of the magnet? (b) What torque is required to hold this magnet at an angle 30° with a uniform field of flux density 0.15 T?

Sol. Here, $m = 48 \text{ Am}$; $2l = 25 \text{ cm} = 0.25 \text{ m}$

(a) Magnetic moment,

$$M = m \times 2l = 48 \times 0.25 = 12 \text{ Am}^2$$

(b) Here, $\theta = 30^\circ$; $B = 0.15 \text{ T}$

$$\text{Now, } \tau = MB \sin \theta = 12 \times 0.15 \times \sin 30^\circ = 0.9 \text{ N m}$$

POTENTIAL ENERGY OF A BAR MAGNET PLACED IN A MAGNETIC FIELD

Let a bar magnet of dipole moment M be placed in a uniform magnetic field of strength B , such that the magnet makes an angle θ with the direction of the field.

Then, magnitude of the torque acting on the dipole is given by

$$\tau = MB \sin \theta$$

This torque tends to align the magnet along the direction of the field. If the magnet is rotated against the action of this torque, work has to be done. Suppose that the magnet is rotated through an infinitesimally small angle $d\theta$ under the action of the constant torque τ . Then, small amount of work done in rotating the magnet through an infinitesimally small angle $d\theta$,

$$dW = \tau d\theta = MB \sin \theta d\theta$$

If the magnet is rotated from initial position $\theta = \theta_1$ to the final position $\theta = \theta_2$, then the total work done is given by

$$\begin{aligned} W &= \int_{\theta_1}^{\theta_2} MB \sin \theta d\theta = MB \int_{\theta_1}^{\theta_2} \sin \theta d\theta = MB [-\cos \theta]_{\theta_1}^{\theta_2} \\ &= -MB (\cos \theta_2 - \cos \theta_1) = MB (\cos \theta_1 - \cos \theta_2) \end{aligned}$$

The work done in rotating the magnet is stored inside the magnet as its potential energy (U). Thus, potential energy of the magnet inside the magnetic field,

$$U = MB(\cos\theta_1 - \cos\theta_2) \quad \dots(i)$$

Suppose that the magnet is initially perpendicular to the direction of the magnetic field i.e. $\theta_1 = 90^\circ$. Then, potential energy of the magnet in any position making angle θ with the direction of the field can be obtained by setting $\theta_1 = 90^\circ$ and $\theta_2 = \theta$ in Eq. (i) i.e.

$$U = MB(\cos 90^\circ - \cos\theta) \quad \dots(ii)$$

$$\text{or } U = -MB \cos\theta$$

$$\text{In vector notation, } U = -\vec{M} \cdot \vec{B} \quad \dots(iii)$$

Special cases: Let us find the potential energy of the dipole in following cases:

- (i) When $\theta = 0^\circ$, $U = -MB \cos 0^\circ = -MB$ (Minimum)
- (ii) When $\theta = 90^\circ$, $U = -MB \cos 90^\circ = 0$
- (iii) When $\theta = 180^\circ$, $U = -MB \cos 180^\circ = MB$ (Maximum)

Thus, a magnetic dipole possesses minimum potential energy when $\vec{M}$ and $\vec{B}$ are like parallel and maximum potential energy when $\vec{M}$ and $\vec{B}$ are antiparallel.

ILLUSTRATION 3.8

Calculate the work done in rotating a bar magnet of magnetic moment 3 J T^{-1} through an angle of 60° from its position along a magnetic field of strength $0.34 \times 10^{-4} \text{ T}$.

Sol. Here, $M = 3 \text{ J T}^{-1}$; $B = 0.34 \times 10^{-4} \text{ T}$; $\theta_1 = 0^\circ$; $\theta_2 = 60^\circ$

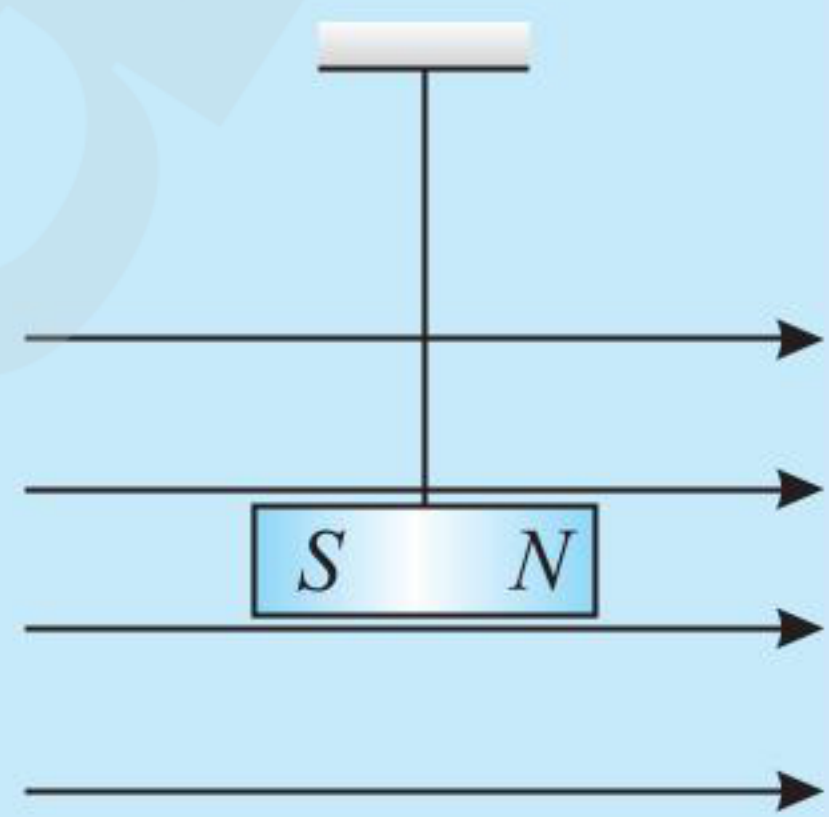
Now, work done to rotate the magnet from the position θ_1 to θ_2 is given by

$$W = MB(\cos\theta_1 - \cos\theta_2)$$

$$\begin{aligned} \therefore W &= 3 \times 0.34 \times 10^{-4} (\cos 0^\circ - \cos 60^\circ) \\ &= 3 \times 0.34 \times 10^{-4} (1 - 0.5) \\ &= 5.1 \times 10^{-5} \text{ J} \end{aligned}$$

ILLUSTRATION 3.9

A magnetic dipole of magnetic moment M is suspended by a string in a uniform horizontal magnetic field as shown in figure. If in horizontal plane this dipole is slightly tilted and released, show that it will execute simple harmonic motion and find its oscillation period. Consider the dipole as a uniform rod of mass m and length l .



Sol. If dipole is tilted from its equilibrium position by a small angle θ , in horizontal plane, restoring torque on magnetic dipoles is given as

$$\tau_R = MB \sin\theta \approx MB\theta$$

The angular acceleration of dipole is given as

$$\alpha = -\frac{\tau_R}{I}$$

$$\Rightarrow \alpha = -\frac{MB}{(ml^2/12)}\theta$$

$$\Rightarrow \alpha = -\frac{12MB}{ml^2}\theta \quad \dots(i)$$

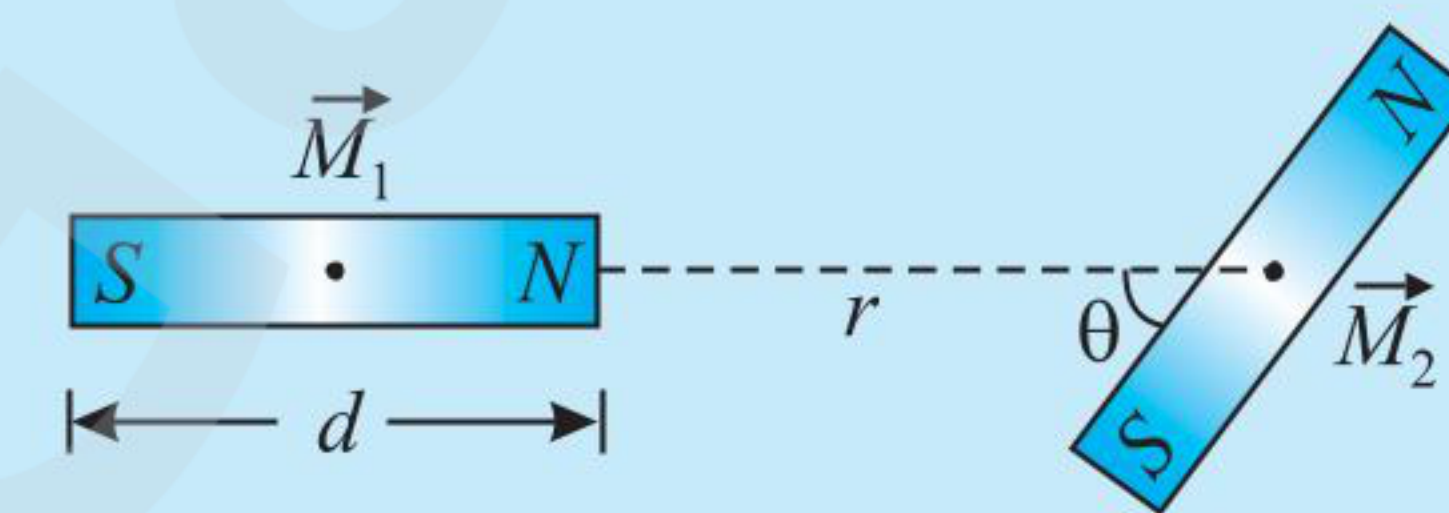
Equation (i) shows that restoring angular acceleration of the dipole is directly proportional to the angular displacement hence it will execute SHM. Comparing this with angular acceleration of standard angular SHM equation given as $\alpha = -\omega^2\theta$, we get

$$\omega = \sqrt{\frac{12MB}{ml^2}}$$

$$\Rightarrow T = 2\pi\sqrt{\frac{ml^2}{12MB}}$$

ILLUSTRATION 3.10

Figure shows two small bar magnets having dipole moments M_1 and M_2 placed at separation r . Find the magnetic interaction energy of this system of dipoles for $d \ll r$.



Sol. Magnetic induction at the location of M_2 due to M_1 is given as

$$B_1 = \frac{\mu_0 M_1}{2\pi r^3}$$

Interaction energy of M_2 in the field of M_1 is given as

$$U = -\vec{M}_2 \cdot \vec{B}_1$$

$$\Rightarrow U = -M_2 B_1 \cos\theta = -\frac{\mu_0 M_1 M_2}{2\pi r^3} \cos\theta$$

ILLUSTRATION 3.11

A magnetic dipole is free to rotate in a uniform magnetic field. For what orientation of the magnet w.r.t. the magnetic field, (a) torque is maximum, (b) potential energy is maximum and (c) rate of change of torque with angle of orientation is maximum?

Sol.

(a) We know, $\tau = MB \sin\theta$

Therefore, torque on the dipole will be maximum, when $\sin\theta$ is maximum i.e. equal to 1 or $\theta = 90^\circ$.

(b) Also $U = -MB \cos\theta$

The maximum value of the potential energy of the dipole is equal to MB . Therefore, potential energy of the dipole will be maximum, when $\cos\theta$ is equal to -1 . i.e. $\theta = 180^\circ$.

Now, rate of change of torque with angle of orientation,

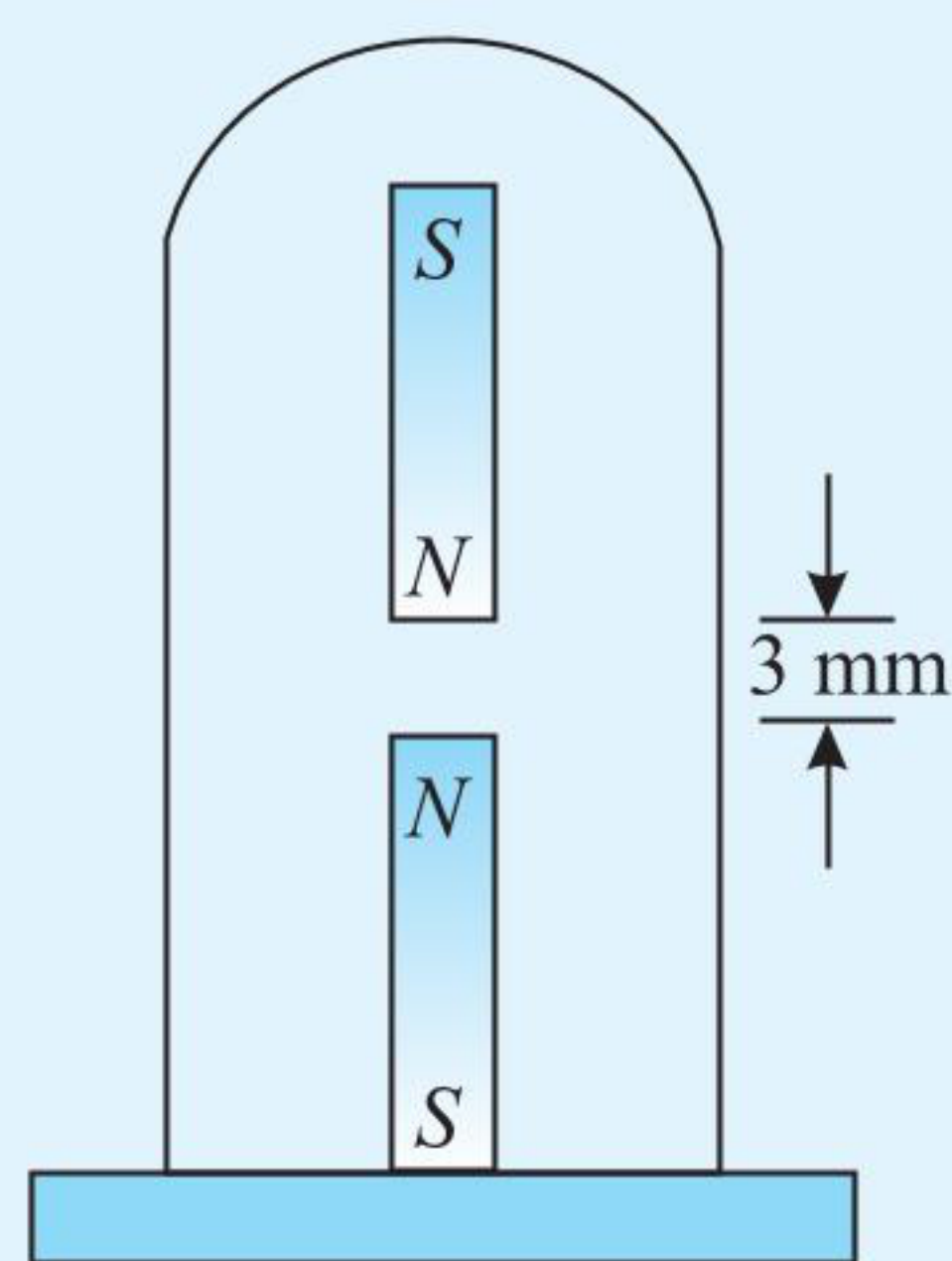
$$\frac{d\tau}{d\theta} = \frac{d}{d\theta} (MB \sin\theta) = MB \cos\theta$$

Therefore, $\frac{d\tau}{d\theta}$ will be maximum, when $\cos\theta$ is

maximum, i.e., equal to 1 or $\theta = 0^\circ$.

CONCEPT APPLICATION EXERCISE 3.1

- Two magnetic poles, one of which is four times stronger than the other, exert a force of 5 gf on each other, when placed at distance of 10 cm in air. Find the strength of each pole.
- Two like magnetic poles of strength 25 Am and 64 Am are situated 1.0 m apart in air. At what point on the line, joining the two poles the magnetic field will be zero?
- Two identical magnets with a length 10 cm and weight 50 gf each are arranged freely with their like poles facing in a vertical glass tube as shown in figure.



- The upper magnet hangs in the air above the lower one so that the distance between the nearest poles of the magnets is 3 mm. Determine the pole strength of the poles of these magnets.
- The electron in the hydrogen atom is moving with a speed of $2.3 \times 10^6 \text{ m s}^{-1}$ in an orbit of radius 0.53 Å. Calculate the magnetic moment of the revolving electron.
 - Two thin bar magnets of pole strengths 4 Am and 7 Am and lengths 0.21 m and 0.12 m respectively are placed at right angles to each other with the N-pole of the first touching the S-pole of the second. Find the magnetic moment of the system.
 - Two short bar magnets of magnetic moments 96 Am^2 and 81 Am^2 are placed along mutually perpendicular straight lines meeting at a point P. Find the magnitude and direction of magnetic field at point P, if it lies at distances of 0.4 m and 0.3 m respectively from the centres of the two magnets.
 - A bar magnet with poles 25 cm apart and pole strength 14.4 Am rests with its centre on a frictionless pivot. It is held in equilibrium at 60° to a uniform magnetic field of induction 0.25 T by applying a force F at right angle to its axis, 10 cm from its pivot. Calculate F . What will happen if the force is removed?
 - A small coil of radius 0.002 m is placed on the axis of a magnet of magnetic moment 10^5 JT^{-1} and length 0.1 m at a distance of 0.15 m from the centre of the magnet. The plane of the coil is perpendicular to the axis of the magnet. Find the force on the coil, when the current of 2.0 A is passed through it.

ANSWERS

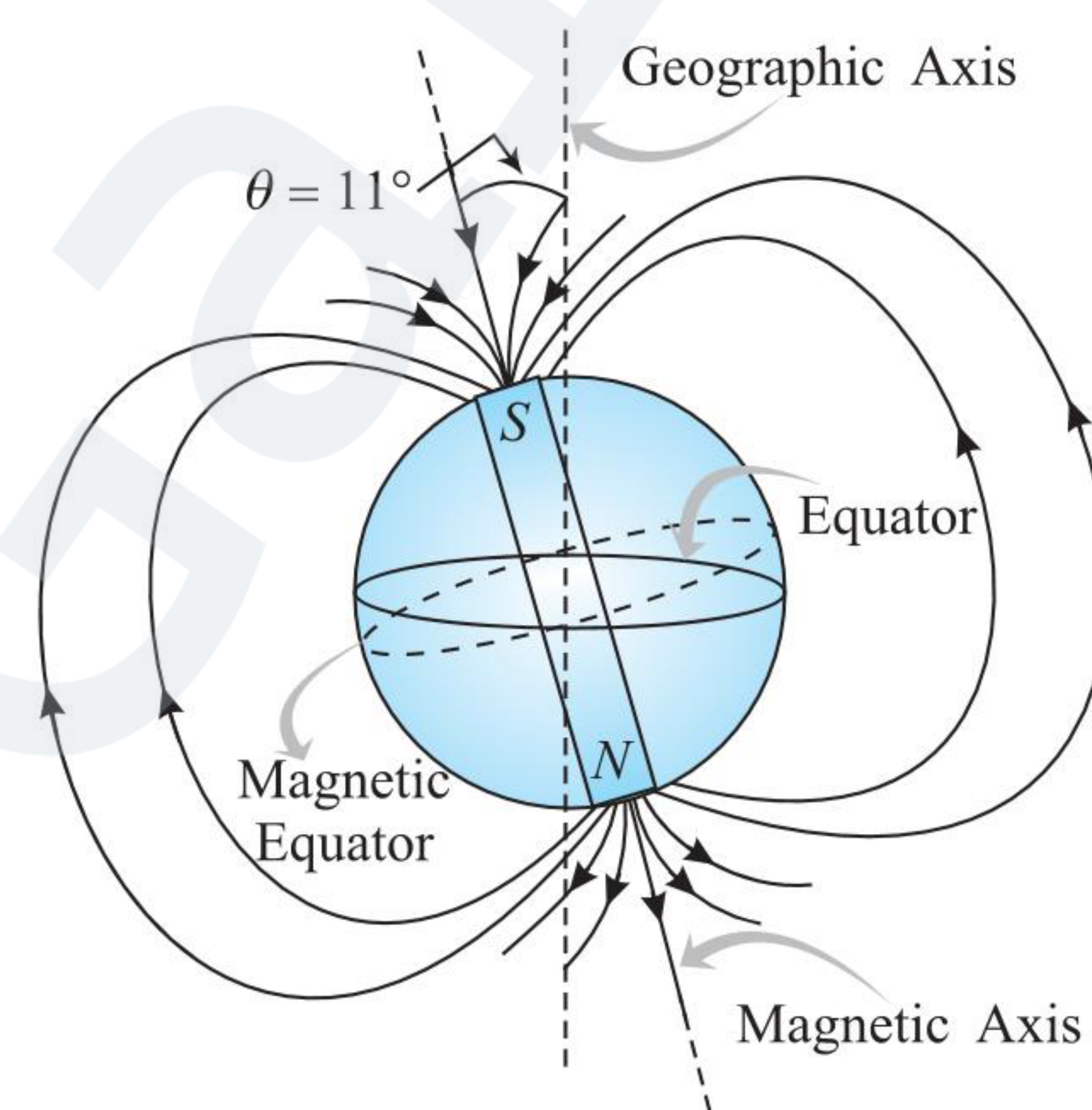
- 35 Am, 140 Am
- Either at 0.385 m from pole of 25 Am (towards 64 Am) or at 1.667 m from pole of 25 Am (on the other side of 64 Am)
- 6.64 Am 4. $9.75 \times 10^{-24} \text{ Am}^2$ 5. 1.19 Am^2
- $3\sqrt{5} \times 10^{-4} \text{ T}$ (making an angle of $\tan^{-1}(2)$ with N_1S_1)
- 6.24 N, the bar magnet will tend to align itself along the field
- $F = 4.4 \times 10^{-3} \text{ N}$

EARTH'S MAGNETISM

The branch of physics that deals with the study of Earth's magnetic field is called terrestrial magnetism or geomagnetism.

Whenever a bar magnet is suspended freely (near the equator), its N-pole points towards the geographical north and S-pole towards the geographical south of the earth.

Earth's magnetic field is also called 'geomagnetic field'. The cause of Earth's magnetic field is the rotation of ionized particles in molten core of earth which constitutes the convection currents at the Earth's core and behave like a coil which produces magnetic field in surrounding and it extends from inner core to outer space. The configuration of Earth's magnetic field is shown in figure.



The axis of rotation of earth is called geographic axis and the points, where it passes through the surface of earth, are called geographic poles. The points N and S represent the geographic north and south poles. The great circle on the earth's surface perpendicular to the geographic axis is called equator.

A vertical plane passing through the geographic axis is called the geographic meridian.

The axis of the huge magnet assumed to be lying inside the earth is called magnetic axis of the earth and the points where the magnetic axis cuts the earth's surface are called earth's magnetic poles. The points N' and S' indicate the magnetic north and south of the earth. The great circle on the earth's surface perpendicular to the magnetic axis is called magnetic equator.

A vertical plane passing through the magnetic axis of the earth is called magnetic meridian.

The axis of Earth's magnetic field is slightly tilted from its axis of rotation (geographic axis) at about 11° . This angle changes with time very slowly due to some geophysical factors not in scope of this book. This axis along which Earth's magnetic coil is considered is called 'magnetic axis' of Earth.

The points on Earth surface where this magnetic axis meet are called magnetic poles. Due to the direction of convection current inside the core of Earth its magnetic induction comes out from the geographic south pole and it gets into the Earth at geographic north pole. Thus the magnetic poles of Earth are located near to the opposite geographic poles as shown in figure.

Cause of earth's magnetism: It may be pointed out that the source of earth's magnetism is not a large solid mass of magnetic material located inside the earth. It is guessed that earth's magnetism is due to molten charged metallic fluid. It gives rise

to a current flowing inside the core of the earth. This core has a radius of about 3500 km. Up to a distance of about 3×10^4 km ($\approx$ five earth's radii), the magnetic field is entirely due to earth's magnetism. At larger distances, the motion of the ions influence the earth's magnetic field. The motion of the ions is affected by a thin hot gas expelled by the sun, called solar wind.

Prof. Blackett studied earth's magnetism in detail and suggested that since every substance is made up of charged particles (protons and electrons). Hence, a substance rotating about an axis will become magnetized. The earth's magnetism is also due to the rotation of the earth about its own axis.

There is another theory about the earth's magnetism. The gases in the atmosphere are in the ionized states. The high energy radiation coming from the sun ionizes the atoms in the upper part of the atmosphere. The radioactivity of the atmosphere and cosmic rays also ionize the gases in the atmosphere. Hence, due to rotation of the earth, strong electric currents flow. Due to action of these currents, the earth is magnetized.

The third view regarding earth's magnetism is that the core of the earth is made of iron and nickel. Due to rotation, the earth behaves as a dynamo and becomes magnetized.

MAGNETIC ELEMENTS OF THE EARTH

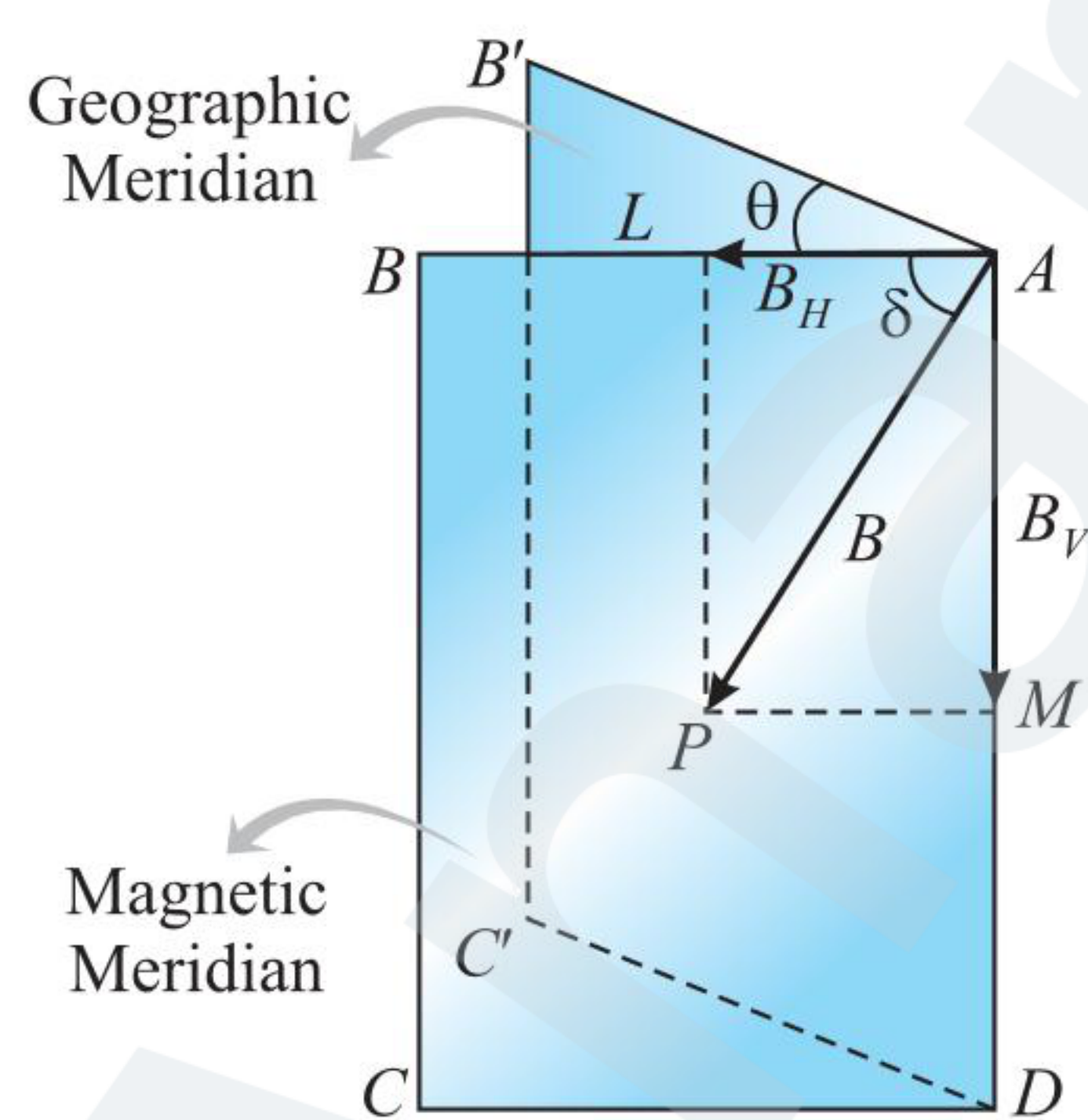
Those quantities which describe the magnetic field of the Earth at a particular place completely in magnitude as well as in direction are known as the magnetic elements at that place.

These are: (a) magnetic declination, (b) dip and (c) horizontal component of the Earth's magnetic field.

Magnetic Declination

Declination at a place is the angle between the geographic meridian and the magnetic meridian. It is denoted by θ .

If $ABCD$ and $AB'C'D$ represent magnetic and geographical meridians, then $\angle BAB' = \theta$ represents the magnetic declination.



Magnetic Inclination or Dip

Dip at a place is defined as the angle made by the direction of the earth's total magnetic field with the horizontal direction. It is denoted by δ .

Suppose that a small bar magnet is suspended freely at point A and it points in direction AP . Then, AP is the direction of the intensity of earth's total magnetic field (B). The line AP lies in magnetic meridian. Consider that AB is a horizontal line in the magnetic meridian. Then, $\angle BAP = \delta$ represents the dip at point A .

Horizontal Component (B_H)

The total intensity of the Earth's magnetic field at any point can be resolved into two rectangular components, one along the horizontal and the other along the vertical direction.

The component of the resultant intensity of the Earth's magnetic field in the horizontal direction is called its horizontal component and is denoted by B_H .

The component of the resultant intensity of the Earth's magnetic field in the vertical direction is called its vertical component and is denoted by B_v .

In figure, the resultant intensity, i.e., B along AP has been resolved into two rectangular components. Clearly, the horizontal component along AB , i.e.,

$$B_H = B \cos \delta \quad \dots(i)$$

and the vertical component along AD , i.e.,

$$B_v = B \sin \delta \quad \dots(\text{ii})$$

From Eqs. (i) and (ii), $\frac{B_V}{B_H} = \frac{B \sin \delta}{B \cos \delta} = \tan \delta$

$$\begin{aligned} \text{and } B_H^2 + B_V^2 &= B^2 \cos^2 \delta + B^2 \sin^2 \delta \\ &= B^2 (\cos^2 \delta + \sin^2 \delta) = B^2 \quad \dots(\text{iii}) \end{aligned}$$

$$\text{or } B = \sqrt{B_H^2 + B_V^2} \quad \dots(\text{iv})$$

Important Points:

- At a place on equator, $B_H = B$ and as such $B \cos \delta = B$ or $\cos \delta = 1$ or $\delta = 0^\circ$.
- Thus, at the equator; $\delta = 0^\circ$, $B_H = B$ and $B_V = 0$.
- At the poles, $B_V = B$ and as such $B \sin \delta = B$ or $\sin \delta = 1$ or $\delta = 90^\circ$. Thus, at the poles; $\delta = 90^\circ$, $B_H = 0$, $B_V = B$.
- The lines drawn through different places having same declination are called **isogonic lines**. A line which passes through places having zero declination is called **agonic line**.
- The lines which pass through different places having same dip are called **isoclinic lines**. A line which passes through places having zero dip is called **aclinic line**.
- The lines drawn through places having the same value of B_H are called **isodynamic lines**.

ILLUSTRATION 3.12

The vertical component of earth's magnetic field at a place is $\sqrt{3}$ times the horizontal component. What is the value of angle of dip at this place?

Sol. Here, $B_V = \sqrt{3} B_H$

$$\text{Now, } \tan \delta = \frac{B_V}{B_H} = \frac{\sqrt{3} B_H}{B_H} = \sqrt{3}$$

or $\delta = 60^\circ$

ILLUSTRATION 3.13

A compass needle whose magnetic moment is 60 Am^2 is pointing geographical north at a certain place whereas the horizontal component of the Earth's magnetic field is $40 \mu\text{Wb/m}^2$ and experiences a torque of $1.2 \times 10^{-3} \text{ Nm}$. What is the declination of the place?

Sol. Here $m = 60 \text{ Am}^2$,

$$B_H = 40 \mu\text{Wb/m}^2 \\ = 40 \times 10^{-6} \text{ Wb/m}^2, \tau = 1.2 \times 10^{-3} \text{ Nm}$$

As $\tau = mB \sin \theta$,

$$\sin \theta = \frac{\tau}{mB_H} = \frac{1.2 \times 10^{-3} \text{ Nm}}{(60 \text{ Am}^2)(40 \times 10^{-6} \text{ Wb/m}^2)} = 0.5$$

Thus, $\theta = 30^\circ$

Here $\theta = 30^\circ$, being the angle between geographic north and magnetic north, is the declination of the place as in stable equilibrium a compass needle points along magnetic north.

ILLUSTRATION 3.14

The horizontal and vertical components of Earth's field at a place are 0.22 G and 0.38 G respectively. Calculate the angle of dip and resultant intensity of Earth's field.

Sol. Here $B_H = 0.22 \text{ G}$, $B_V = 0.38 \text{ G}$.

$$\text{As } \tan \delta = \frac{B_V}{B_H} = \frac{0.38}{0.22} = 1.73, \delta = 60^\circ$$

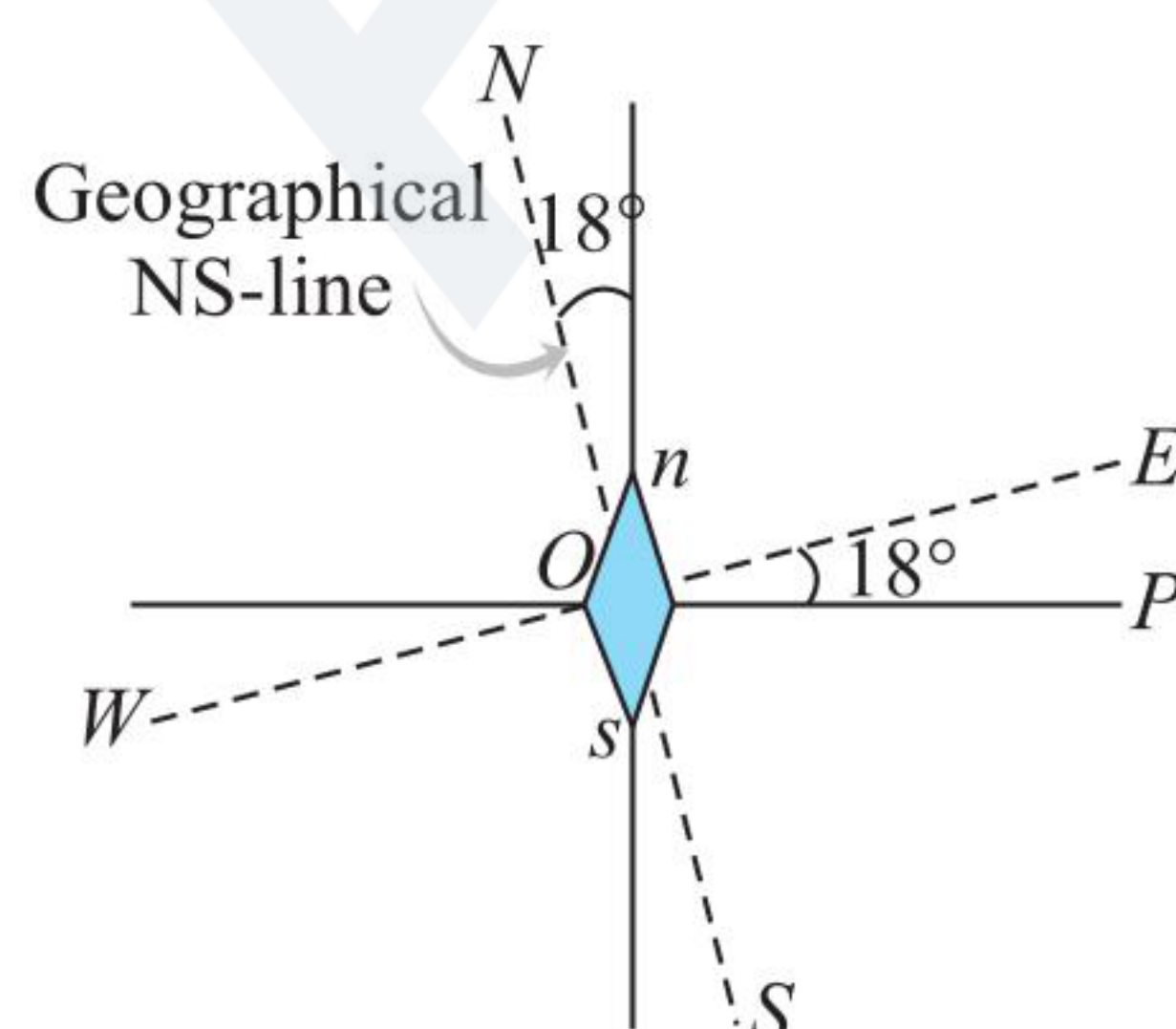
Resultant intensity,

$$B = \sqrt{B_H^2 + B_V^2} = \sqrt{(0.22)^2 + (0.38)^2} = 0.44 \text{ G}$$

ILLUSTRATION 3.15

A ship is sailing due east according to Mariner's compass. If the declination of that place is 18° east of north, what is the true direction of the ship?

Sol. The compass needle NS points along the magnetic NS-line and the ship is sailing along OP i.e. towards east (as indicated by compass needle). As the declination of that place is 18° east of north, the geographical NS line is as shown by dotted NS line (figure).



Thus, the true direction of ship is 18° south of east.

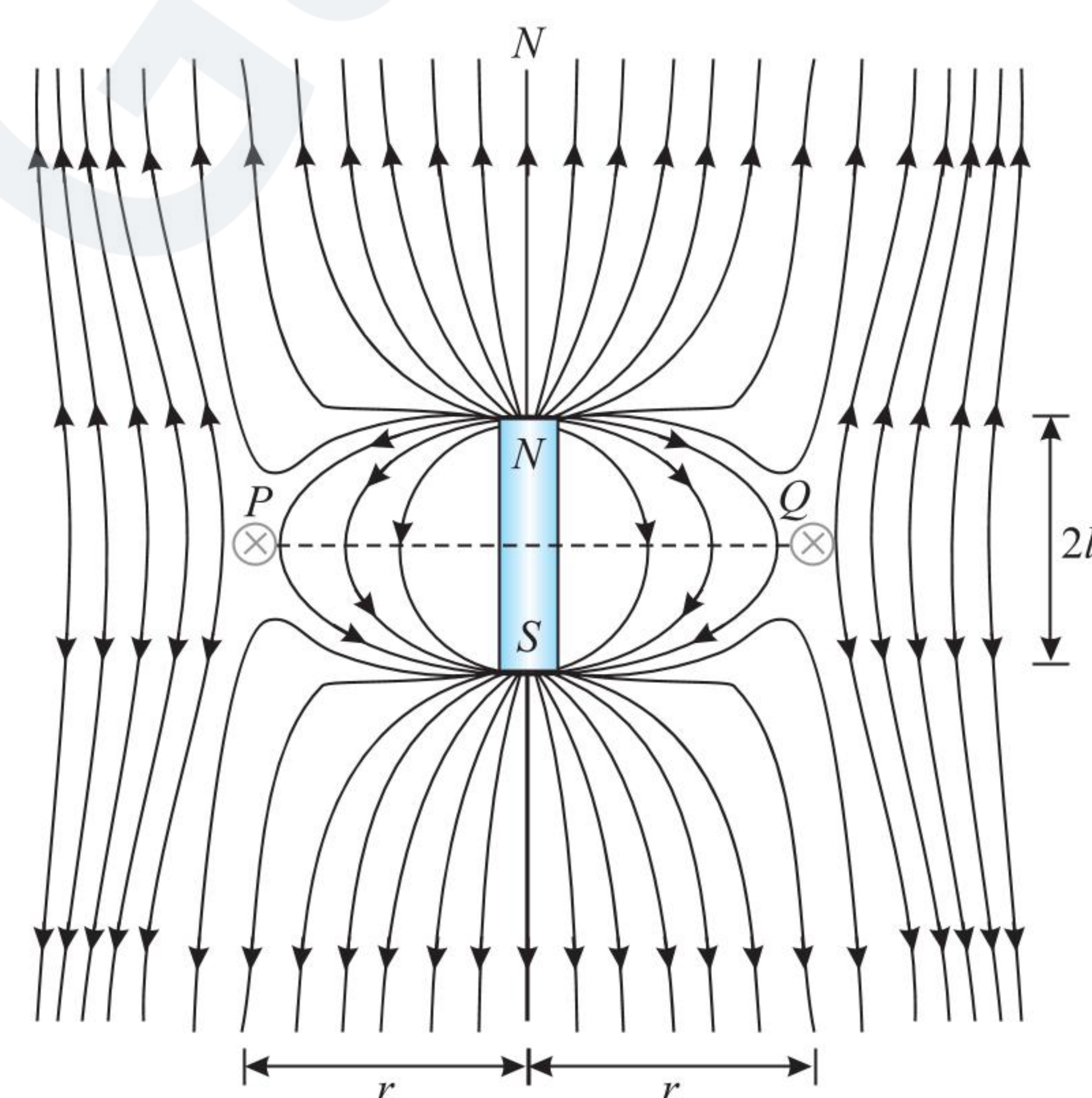
NEUTRAL POINT AND TO FIND MAGNETIC MOMENT OF A BAR

Neutral point: A neutral point in the magnetic field of a bar magnet is that point where the field due to the magnet is completely neutralized by the horizontal component of earth's magnetic field. Thus, at the neutral point, the magnetic field due to the bar magnet (B) is just equal and opposite to the horizontal component of earth's magnetic field (B_H) i.e.

$$B = B_H \quad \dots(i)$$

To find magnetic moment of a bar magnet: The magnetic moment of a bar magnet can be found by plotting magnetic field of the magnet, locating the neutral points and then using Eq. (i).

(a) **When magnet is placed with its north pole towards north of the earth:** If we plot the magnetic field due to a small bar magnet by placing its north pole towards north of the earth, then plot of the magnetic field of the magnet (which is superposed by earth's magnetic field) will be as shown in figure.



The horizontal component of earth's magnetic field and the magnetic field due to the bar magnet cancel out at the two cross-marks on the equatorial line of the magnet (the normal bisector of the magnet). These points are called neutral points. Let r be distance of a neutral point from the center of the magnet. As the neutral point lies on the equatorial line of the magnet, the magnetic field due to the magnet at the neutral point is given by

$$B_{\text{equi}} = \frac{\mu_0}{4\pi} \cdot \frac{M}{r^3}$$

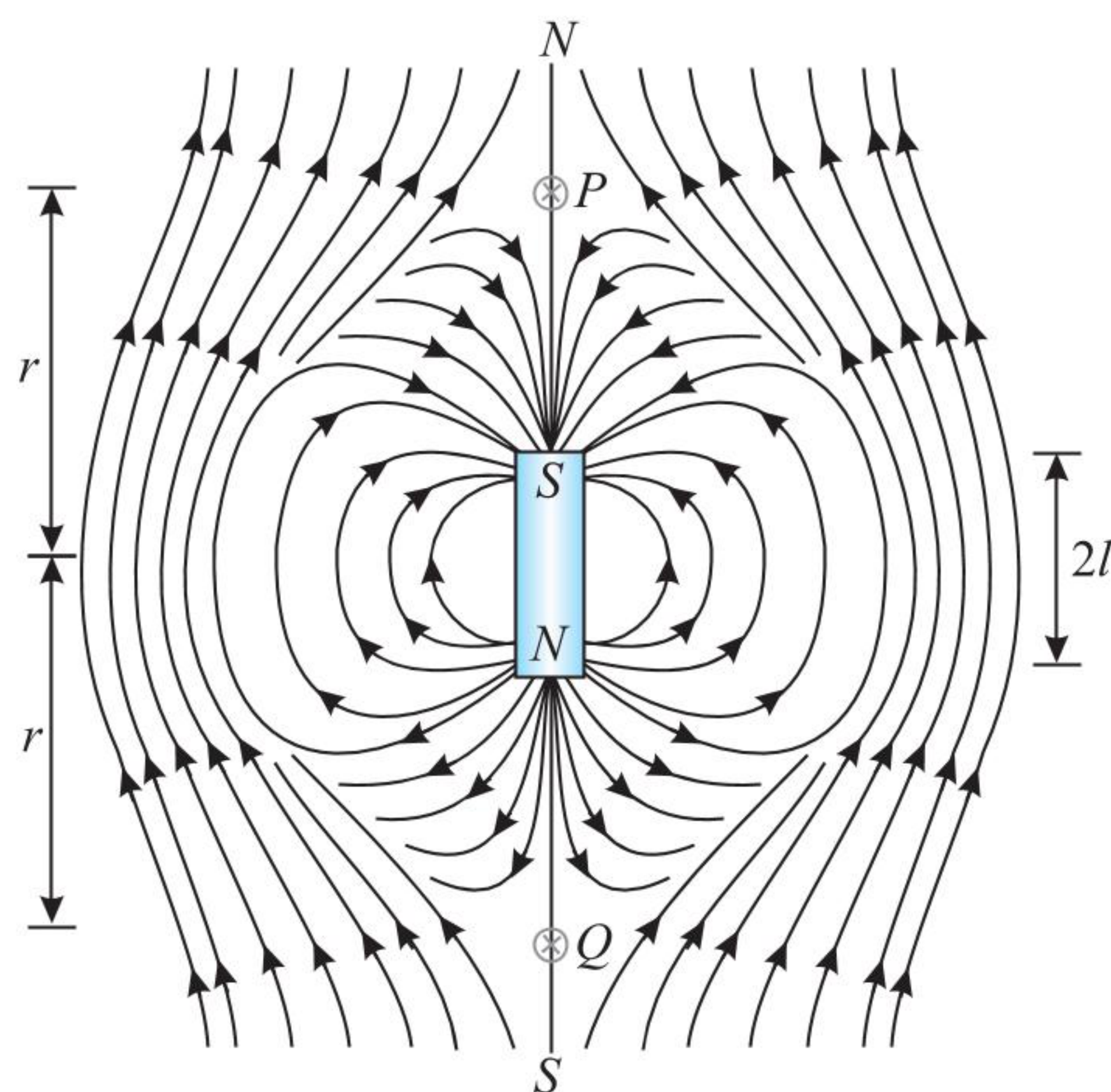
where M is magnetic dipole moment of the magnet.

Since at the neutral point, magnetic field due to the magnet is equal to B_H , we have

$$\frac{\mu_0}{4\pi} \cdot \frac{M}{r^3} = B_H \quad \dots(ii)$$

Knowing r and B_H , the magnetic dipole moment of the bar magnet can be calculated from Eq. (ii).

(b) **When magnet is placed with its south pole towards north of the earth:** The plot of the magnetic field of a bar magnet in this case will be as shown in figure.



In this case, the neutral points are found to be on axial line of the magnet. If neutral point is at a distance r from the centre of the magnet, then magnetic field due to the magnet at the neutral point (point on the axial line of the magnet) is given by

$$B_{\text{axial}} = \frac{\mu_0}{4\pi} \cdot \frac{2M}{r^3}$$

Since at the neutral point, the magnetic field due to the magnet is equal to B_H , we have

$$\frac{\mu_0}{4\pi} \cdot \frac{2M}{r^3} = B_H \quad \dots(iii)$$

Knowing r and B_H , Eq. (iii) can be used to find the magnetic dipole moment of the bar magnet.

Note. From the plot of magnetic field in the two cases i.e. when magnet is placed with its N-pole towards north of earth and when magnet is placed with its S-pole towards north of the earth, it follows that when magnet is turned through 180° , the neutral points turn through 90° .

ILLUSTRATION 3.16

How many neutral points on a horizontal board are there and why, when a magnet is held vertically on the board?

Sol. Let us consider a magnet NS placed vertically on a horizontal board as shown in figure. The neutral point on the horizontal board will be obtained, where the component of the magnetic field (B) due to the magnet in the plane of the board is neutralized, by the horizontal component of the earth's magnetic field (B_H). The directions of magnetic fields B and B_H have been shown at four points P_1 , P_2 , P_3 and P_4 . It follows that the two fields can balance each other only at the point P_2 . Therefore, when a magnet is placed vertically on a the horizontal board, only one neutral point will be obtained.

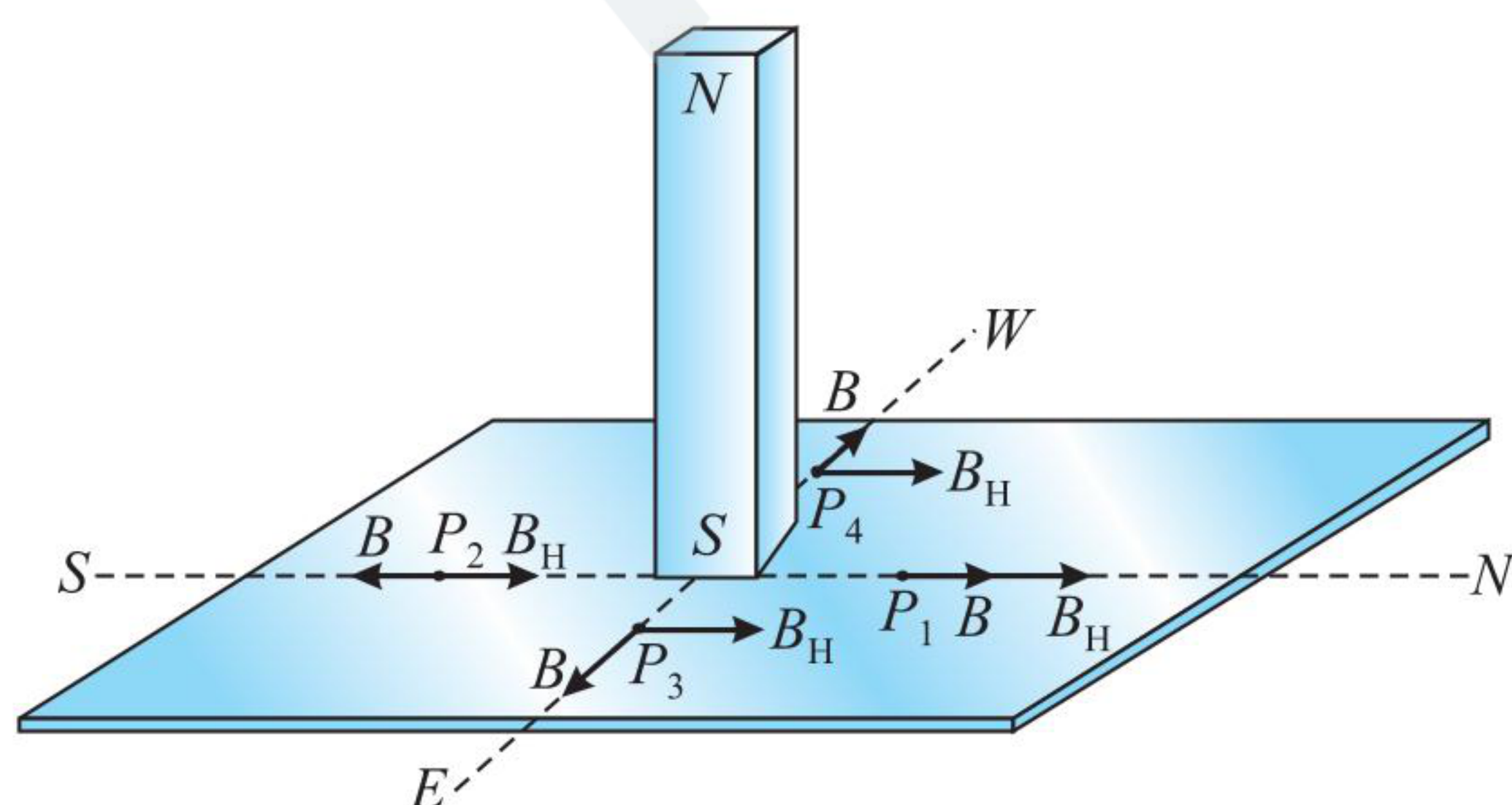


ILLUSTRATION 3.17

A bar magnet 30 cm long is placed in the magnetic meridian with its north pole pointing south. The neutral point is observed at a distance of 30 cm from its one end. Calculate the pole strength of the magnet. Given, horizontal component of earth's field = 0.34 G.

Sol. Here, $2l = 30 \text{ cm}$ or $l = 15 \text{ cm} = 0.15 \text{ m}$;

$$r = 30 \text{ cm} = 0.30 \text{ m},$$

$$B_H = 0.34 \text{ G} = 0.34 \times 10^{-4} \text{ T}$$

When the magnet is placed with its north pole pointing south, neutral point is obtained on its axial line. Therefore, at the neutral point,

$$B_{\text{axial}} = B_H$$

$$\text{or} \quad \frac{\mu_0}{4\pi} \times \frac{2Mr}{(r^2 - l^2)^2} = B_H$$

$$\begin{aligned} \text{or} \quad M &= \frac{4\pi}{\mu_0} \times \frac{B_H (r^2 - l^2)^2}{2r} \\ &= \frac{1}{10^{-7}} \times \frac{0.34 \times 10^{-4} \times (0.30^2 - 0.15^2)^2}{2 \times 0.30} \\ &= \frac{0.34 \times 10^{-4} \times (0.0675)^2}{10^{-7} \times 2 \times 0.30} = 2.582 \text{ Am}^2 \end{aligned}$$

The pole strength of the magnet,

$$m = \frac{M}{2l} = \frac{2.582}{0.30} = 8.607 \text{ Am}$$

ILLUSTRATION 3.18

A bar magnet is 10 cm long. Its magnetic moment is 0.4 Am^2 . It is placed in magnetic meridian with its north pole towards north. If the neutral point is obtained at a distance of 10 cm from the center of the magnet, find the horizontal component of earth's magnetic field.

Sol. Here, $M = 0.4 \text{ Am}^2$; $r = 10 \text{ cm} = 0.1 \text{ m}$;

$$2l = 10 \text{ cm} \text{ or } l = 5 \text{ cm} = 0.05 \text{ m}$$

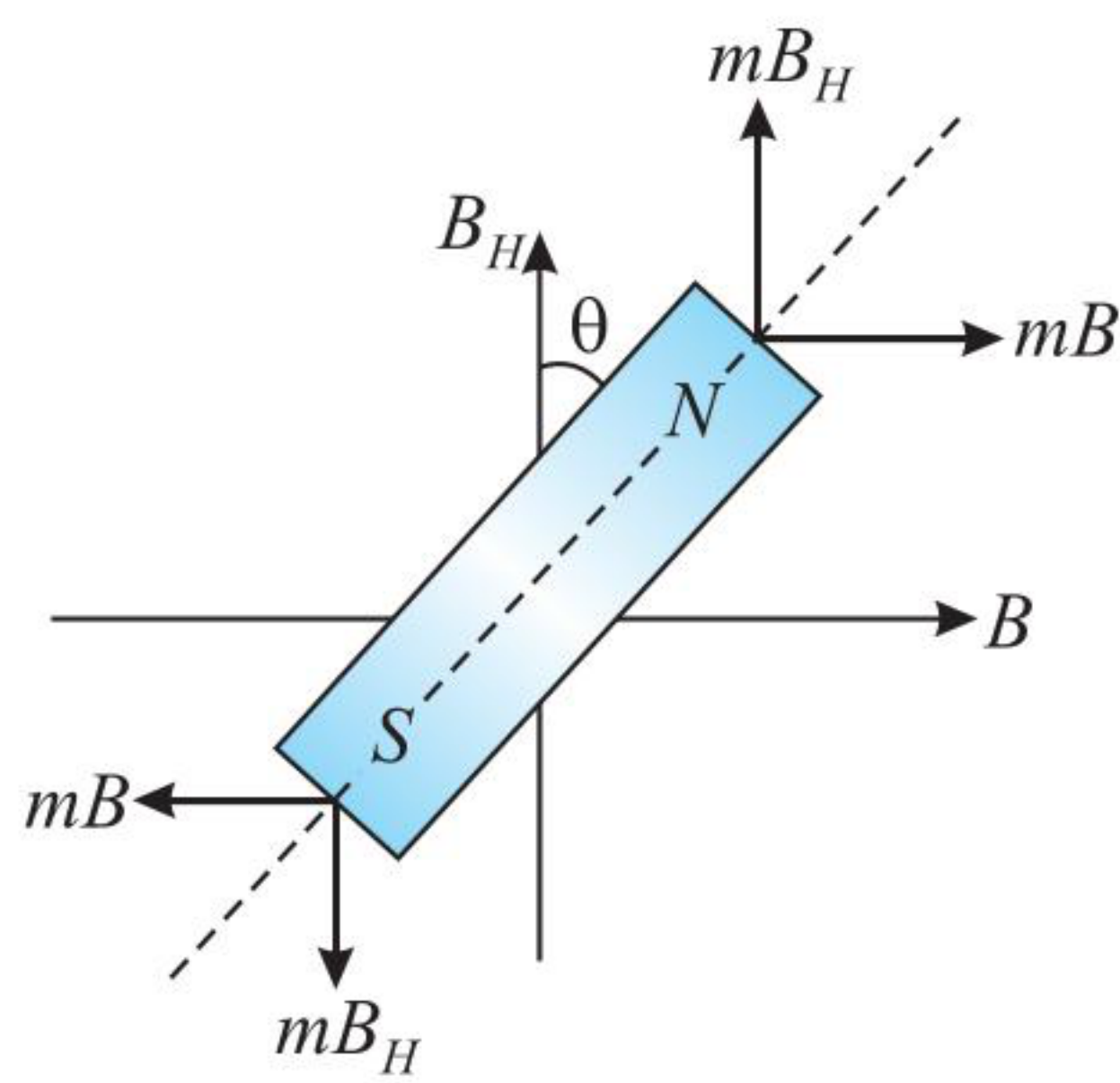
When the magnet is placed with its north pole towards north of earth, the neutral point is obtained on the equatorial line. If B_H is horizontal component of the earth's field, then at the neutral point

$$B_{\text{equi}} = B_H$$

$$\begin{aligned} \text{or} \quad B_H &= \frac{\mu_0}{4\pi} \times \frac{M}{(r^2 + l^2)^{3/2}} \\ &= \frac{10^{-7} \times 0.4}{(0.1^2 + 0.05^2)^{3/2}} = \frac{10^{-7} \times 0.4}{(0.0125)^{3/2}} = \frac{10^{-7} \times 0.4}{0.0014} \\ &= 0.286 \times 10^{-4} \text{ T} \end{aligned}$$

TANGENT LAW

When a small magnet is suspended in two uniform magnetic fields B and B_H which are at right angles to each other, the magnet comes to rest at an angle θ with respect to B_H .



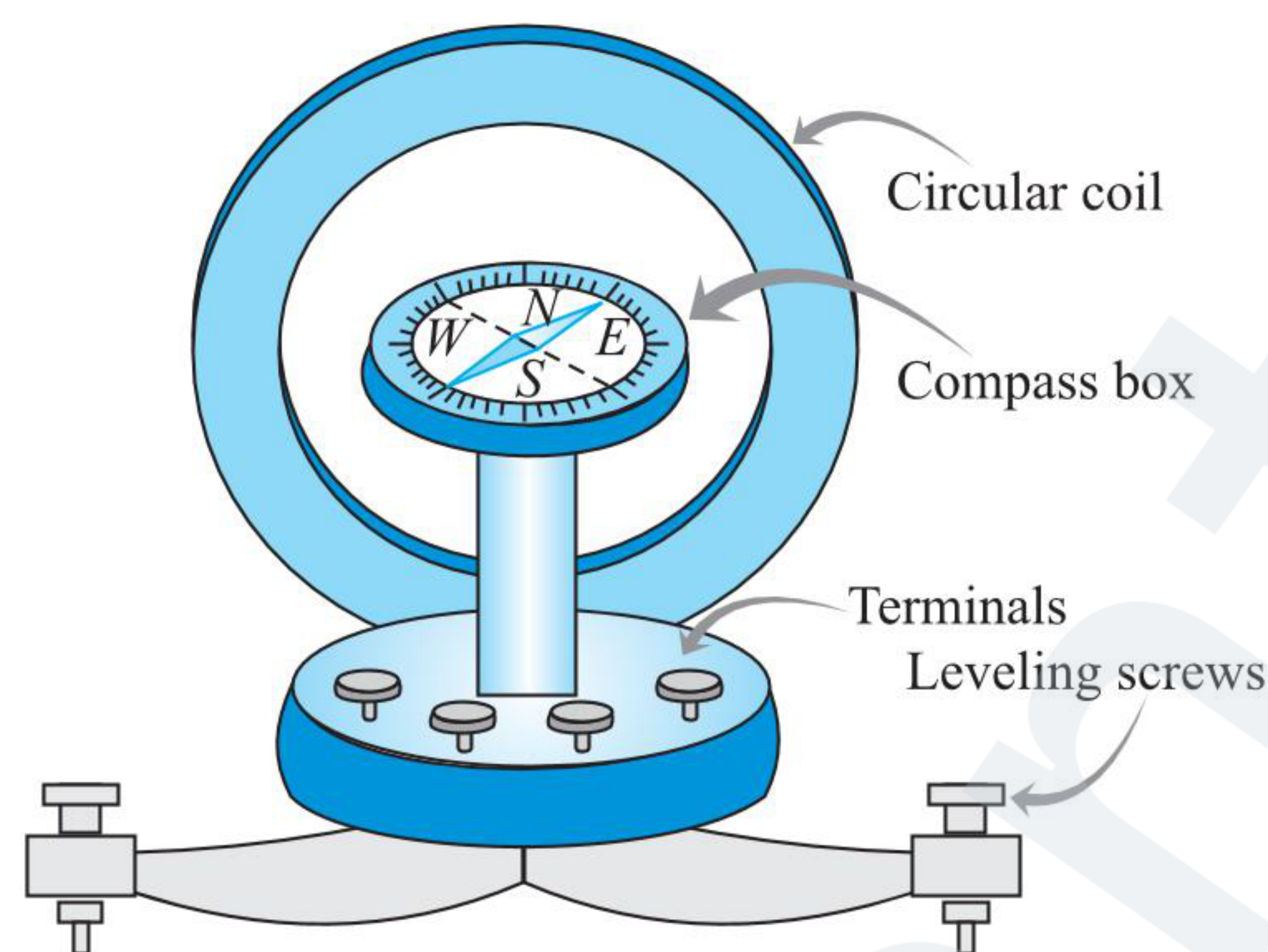
In equilibrium

$$MB_H \sin \theta = MB \sin (90^\circ - \theta)$$

$$\Rightarrow B = B_H \tan \theta. \text{ This is called tangent law.}$$

TANGENT GALVANOMETER

Tangent galvanometer is a device used to find horizontal component of Earth's magnetic induction. It consists of a circular coil (C) of insulated copper wire wound on a vertical circular frame made of nonmagnetic material as ebonite or wood. A small magnetic compass needle is pivoted at the center of the vertical circular frame. When the coil of the tangent galvanometer is kept in magnetic meridian and current passes through any of the coil then the needle at the center gets deflected and comes to an equilibrium position under the action of two perpendicular field: one due to horizontal component of earth and the other due to field (B) set up by the coil due to current.



$$\text{In equilibrium } B = B_H \tan \theta$$

where $B = \frac{\mu_0 n i}{2r}$; n = number of turns, r = radius of coil, i = the current to be measured, θ = angle made by needle from the direction of B_H in equilibrium.

$$\text{Hence } \frac{\mu_0 N i}{2r} = B_H \tan \theta$$

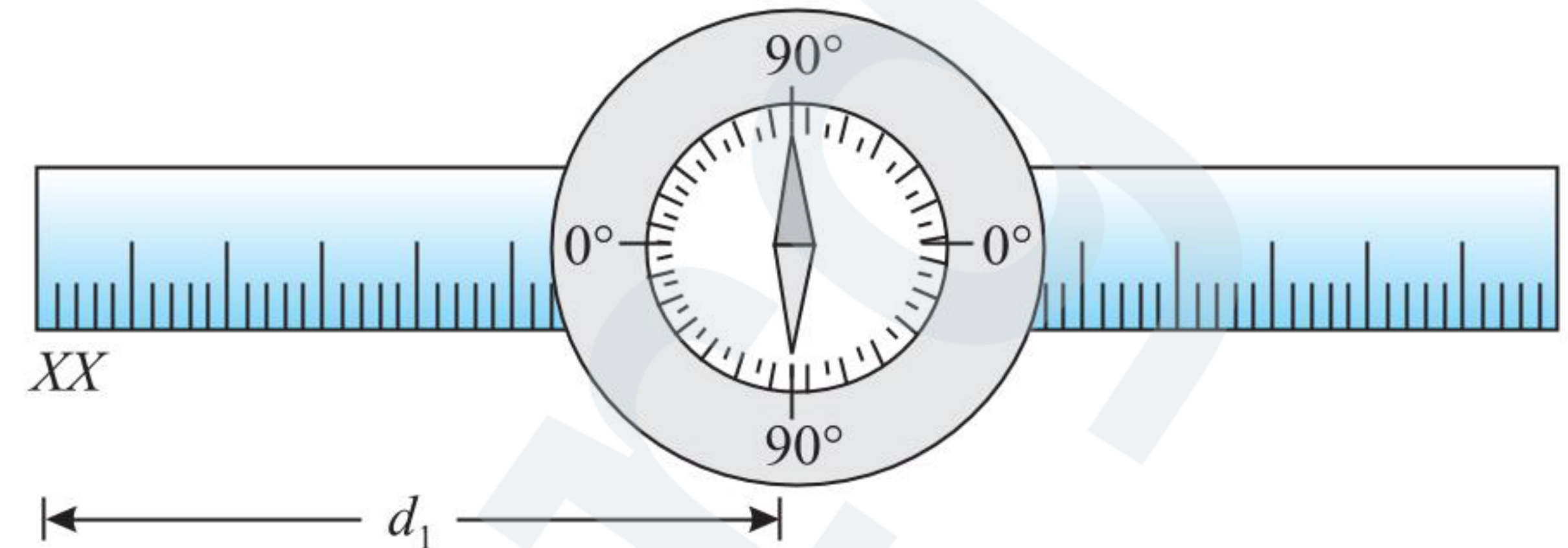
$$\Rightarrow i = k \tan \theta \text{ where } k = \frac{2r B_H}{\mu_0 N} \text{ is called reduction factor.}$$

DEFLECTION MAGNETOMETER

Deflection magnetometer is an experimental setup used to measure the magnetic moment and pole strength of a bar magnet. It is also used to compare the pole strengths of two bar magnets. It's working is based on the principle of tangent law. It consists of a small compass needle, pivoted at the centre of a circular box. The box is kept in a wooden frame having two meter scale fitted

on its two arms. Reading of a scale at any point directly gives the distance of that point from the centre of compass needle.

It consists of a small magnetic compass needle pivoted at the center of a scale as shown in figure. The compass is fixed at the scale in such a way that its circular scale can be rotated which is marked with $0^\circ-90^\circ-0^\circ-90^\circ$ in perpendicular direction, which is kept exactly perpendicular to the scale, as shown in figure.

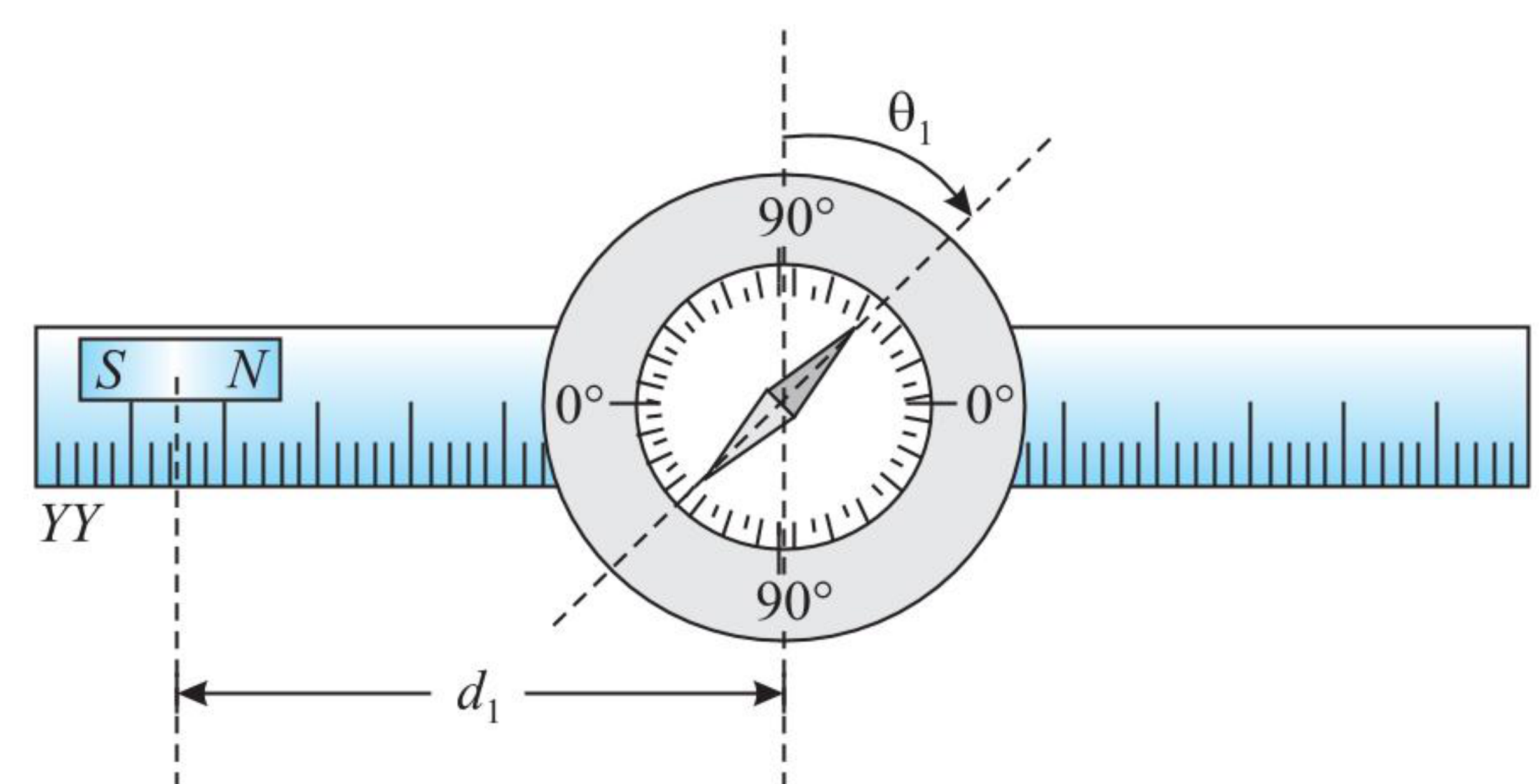


For measuring the magnetic moment of a bar magnet or a magnetic dipole, there are two positions of the magnetometer called 'Tan A' and 'Tan B' positions.

(1) Tan A position: In this position the magnetometer is set perpendicular to magnetic meridian. So that, magnetic field due to magnet, is in axial position and perpendicular to earth's field.

The magnetic dipole of which magnetic moment is to be measured is also placed along the scale line as shown at some distance r from the compass needle. The magnetic induction due to the magnetic dipole at the location of compass is given as

$$B_r = \frac{\mu_0}{4\pi} \cdot \frac{2M}{r^3}$$



If horizontal component of earth's magnetic field is B_H then in Tan A position deflection of compass needle is given as

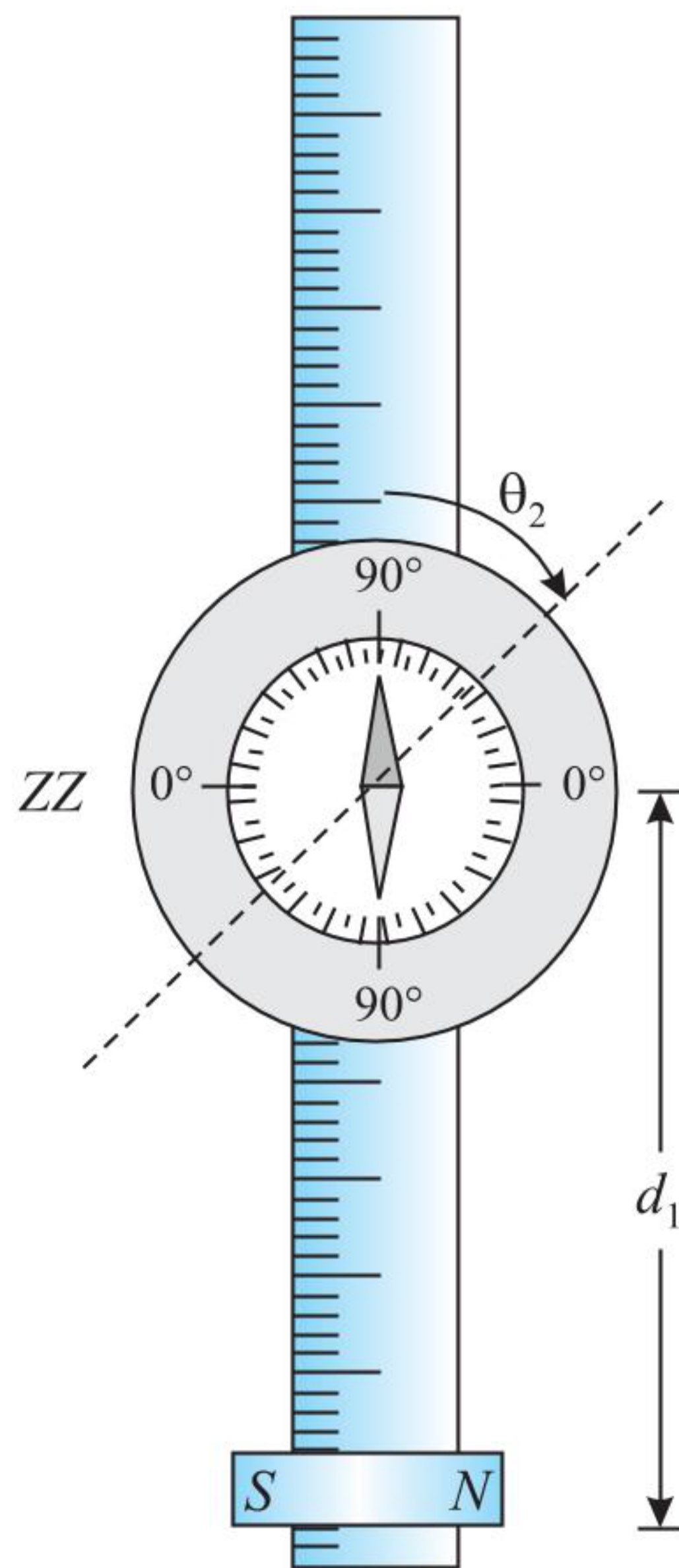
$$\tan \theta = \frac{B_r}{B_H} \quad \dots(i)$$

$$\Rightarrow B_r = B_H \tan \theta \Rightarrow \frac{\mu_0}{4\pi} \cdot \frac{2M}{r^3} = B_H \tan \theta$$

$$\Rightarrow M = \frac{2\pi B_H r^3 \tan \theta}{\mu_0} \quad \dots(ii)$$

(2) Tan B position: The arms of magnetometer are set in magnetic meridian, so that the magnetic field due to magnet is at it's equatorial position. At the same distance from the compass the magnetic dipole is now placed along east-west line perpendicular to the scale as shown in figure. At the location of compass needle the magnetic induction due to the magnetic dipole is given as

$$B_r = \frac{\mu_0}{4\pi} \cdot \frac{M}{r^3}$$



Now again the compass needle deflects under the influence of two mutually perpendicular magnetic fields. So its deflection is given as

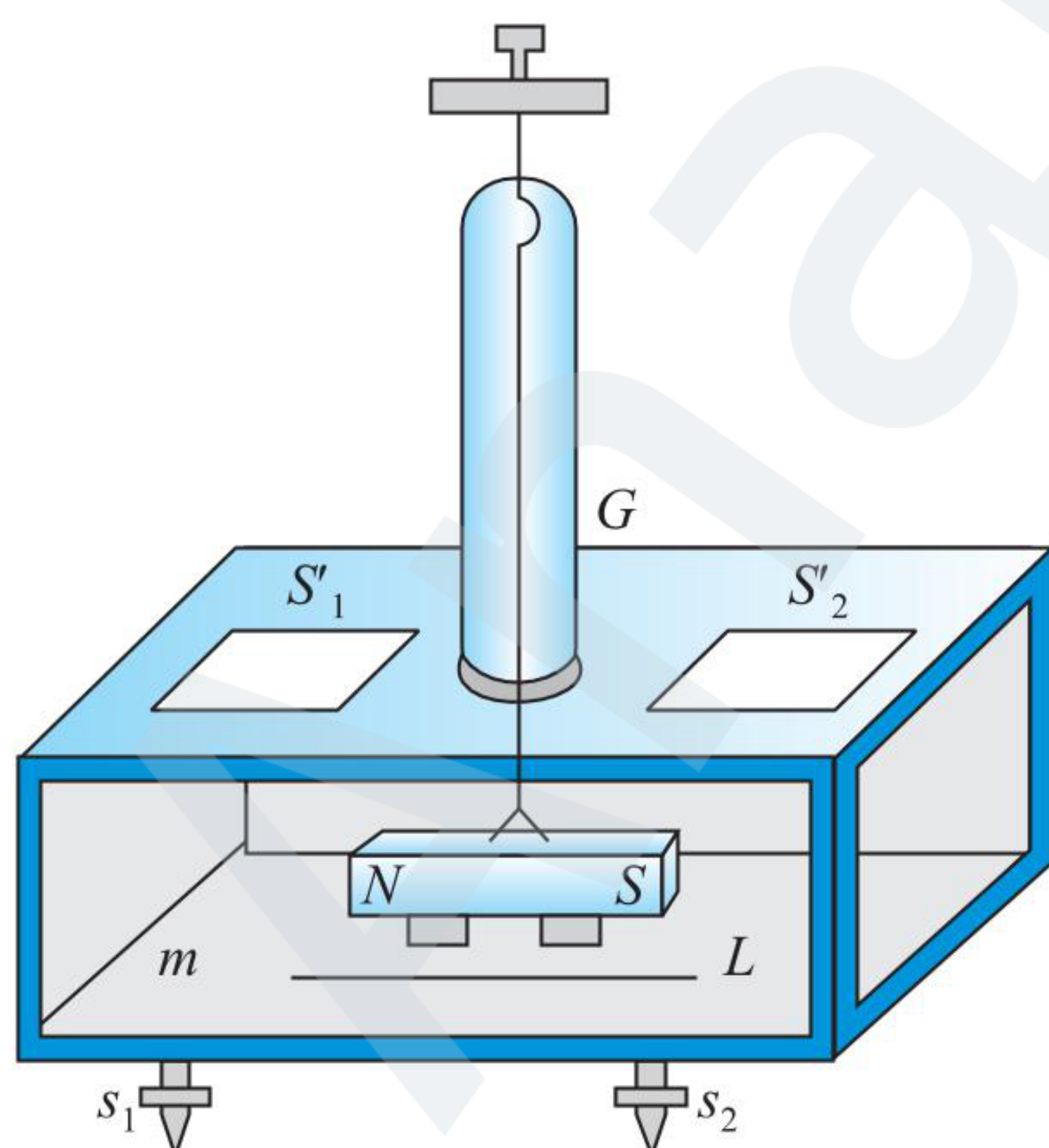
$$\tan \theta = \frac{B_r}{B_H} \quad \dots(iii)$$

$$\Rightarrow B_r = B_H \tan \theta \Rightarrow \frac{\mu_0}{4\pi} \cdot \frac{M}{r^3} = B_H \tan \theta$$

$$\Rightarrow M = \frac{4\pi B_H r^3 \tan \theta}{\mu_0} \quad \dots(iv)$$

VIBRATION MAGNETOMETER

Vibration magnetometer is used for comparison of magnetic moments and magnetic fields. This device works on the principle that whenever a freely suspended magnet in a uniform magnetic field is disturbed from its equilibrium position, it starts vibrating about the mean position.



Time period of oscillation of experimental bar magnet (magnetic moment M) in earth's magnetic field (B_H) is given by the formula. $T = 2\pi \sqrt{\frac{I}{MB_H}}$; where, I = moment of inertia of

$$\text{short bar magnet} = \frac{wL^2}{12} \quad (w = \text{mass of bar magnet})$$

Determination of magnetic moment of a magnet: The experimental (given) magnet is put into vibration magnetometer and its time period T is determined. Now

$$T = 2\pi \sqrt{\frac{I}{MB_H}} \Rightarrow M = \frac{4\pi^2 I}{B_H T^2}$$

Comparison of horizontal components of earth's magnetic field at two places

$$T = 2\pi \sqrt{\frac{I}{MB_H}} ; \text{ since } I \text{ and } M \text{ of the magnet are constant,}$$

$$\text{So } T^2 \propto \frac{1}{B_H} \Rightarrow \frac{(B_H)_1}{(B_H)_2} = \frac{T_2^2}{T_1^2}$$

Comparison of magnetic moment of two magnets of same size and mass

$$T = 2\pi \sqrt{\frac{I}{M \cdot B_H}} ; \text{ Here } I \text{ and } B_H \text{ are constants.}$$

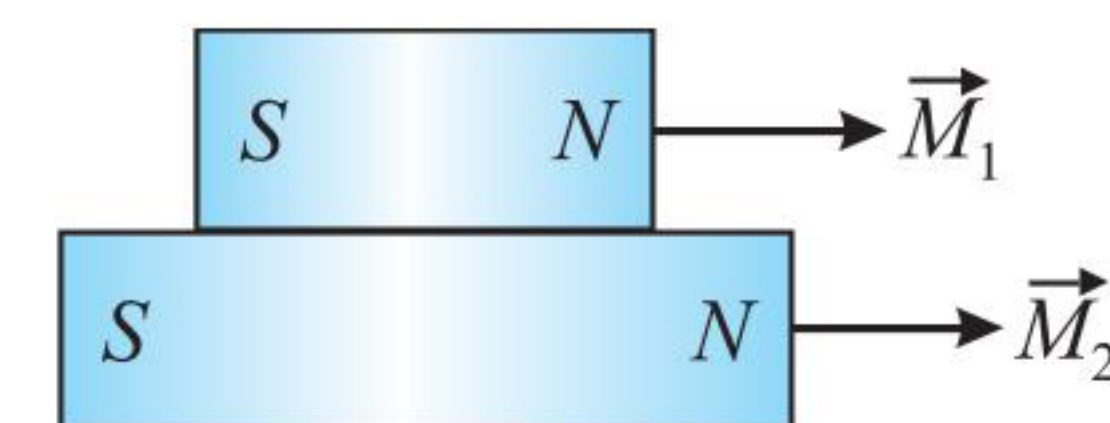
$$\text{So } M \propto \frac{1}{T^2} \Rightarrow \frac{M_1}{M_2} = \frac{T_2^2}{T_1^2}$$

Comparison of magnetic moments by sum and difference method

Sum position

Net magnetic moment $M_s = M_1 + M_2$

Net moment of inertia $I_s = I_1 + I_2$



Time period of oscillation of this pair in earth's magnetic field (B_H)

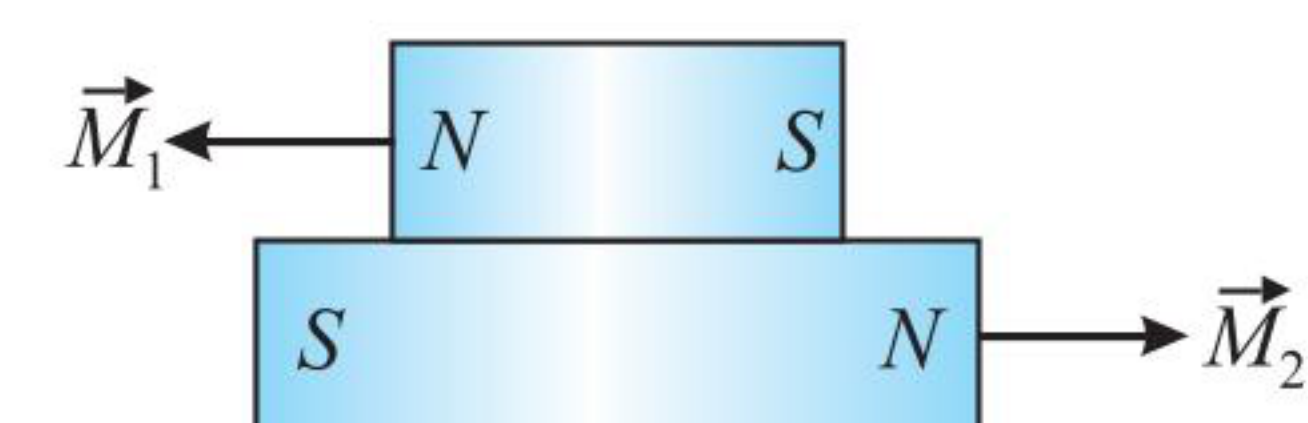
$$T_s = 2\pi \sqrt{\frac{I_s}{M_s B_H}} = 2\pi \sqrt{\frac{I_1 + I_2}{(M_1 + M_2) B_H}} \quad \dots(i)$$

$$\text{Frequency } f_s = \frac{1}{2\pi} \sqrt{\frac{(M_1 + M_2) B_H}{I_s}}$$

Difference position

Net magnetic moment, $M_d = (M_1 - M_2)$

Net moment of inertia $I_d = I_1 + I_2$



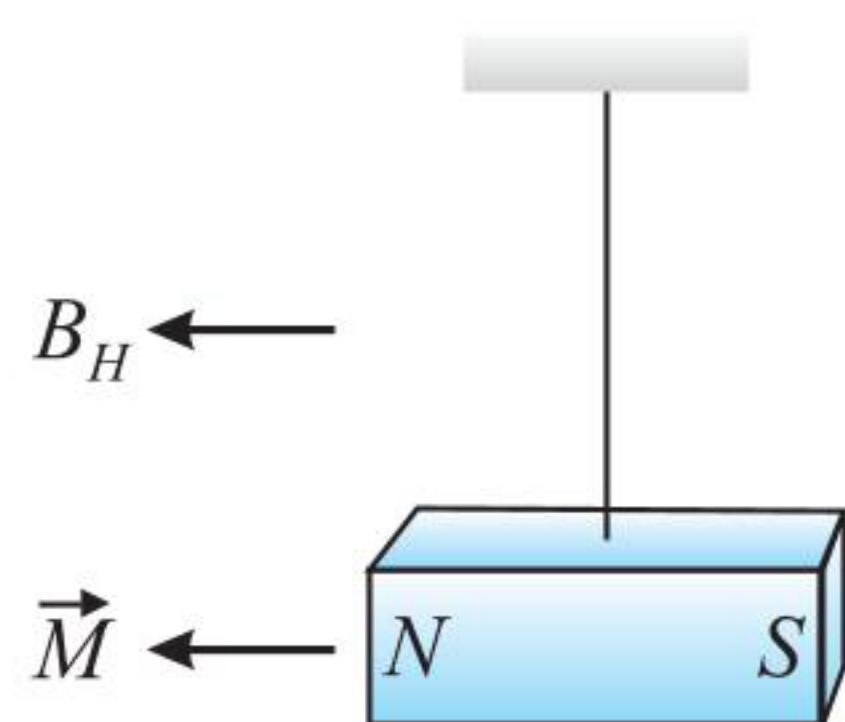
$$\text{and } T_d = 2\pi \sqrt{\frac{I_d}{M_d B_H}} = 2\pi \sqrt{\frac{I_1 + I_2}{(M_1 - M_2) B_H}} \quad \dots(ii)$$

$$\text{and } f_d = \frac{1}{2\pi} \sqrt{\frac{(M_1 + M_2) B_H}{(I_1 + I_2)}}$$

From Eq. (i) and (ii), we get

$$\frac{T_s}{T_d} = \sqrt{\frac{M_1 - M_2}{M_1 + M_2}} \Rightarrow \frac{M_1}{M_2} = \frac{T_d^2 + T_s^2}{T_d^2 - T_s^2} = \frac{f_s^2 + f_d^2}{f_s^2 - f_d^2}$$

To find the ratio of magnetic field: Suppose it is required to find the ratio B/B_H where B is the field created by magnet and B_H is the horizontal component of earth's magnetic field.

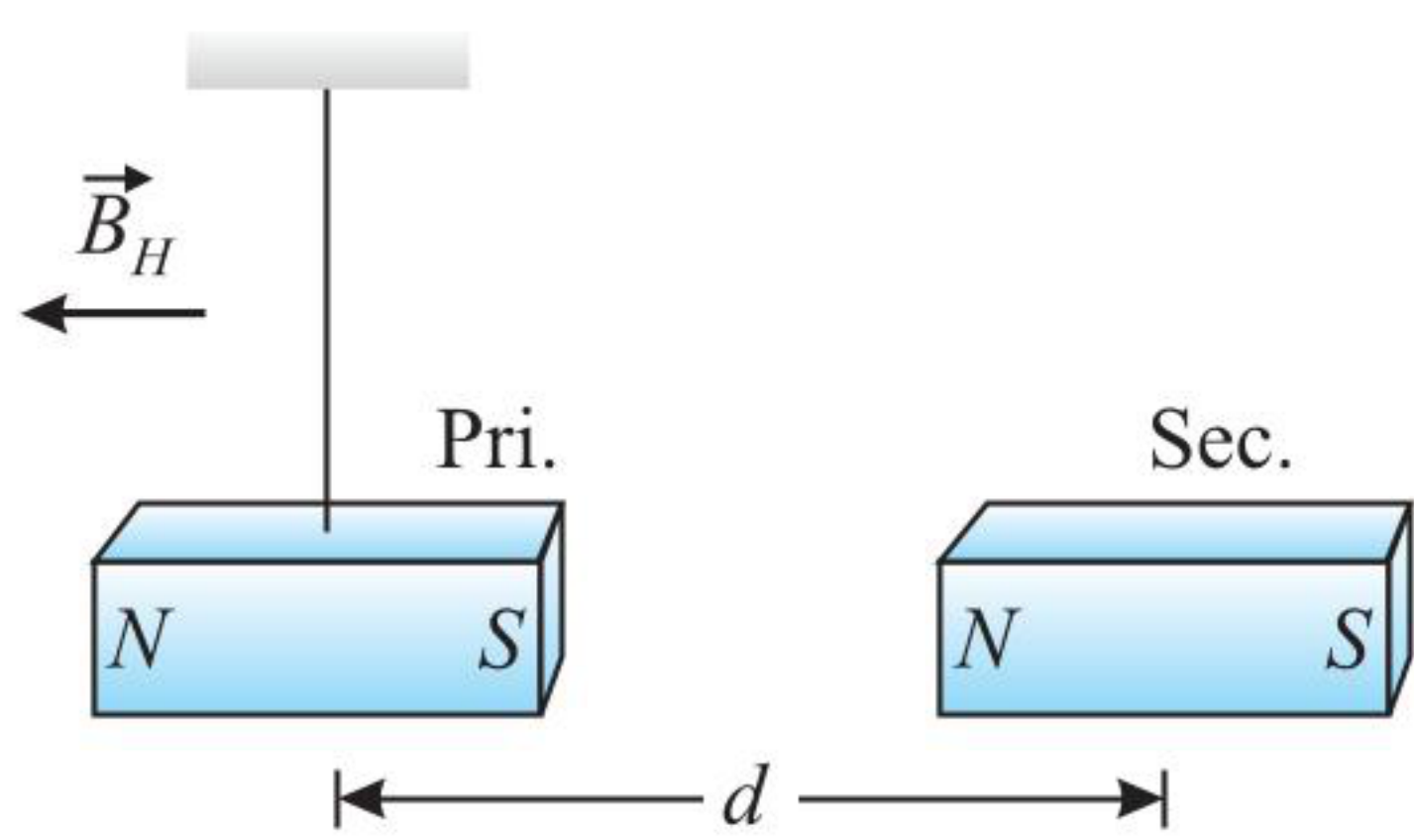


To determine B/B_H a primary (main) magnet is made to first oscillate in earth's magnetic field (B_H) alone and its time period of oscillation (T) is noted.

$$T = 2\pi \sqrt{\frac{I}{MB_H}}$$

and frequency, $f = \frac{1}{2\pi} \sqrt{\frac{MB_H}{I}}$

Now a secondary magnet is placed near the primary magnet. So primary magnet oscillate in a new field which is the resultant of B and B_H and now time period is noted again.



$$T' = 2\pi \sqrt{\frac{I}{M(B + B_H)}}$$

or $f' = \frac{1}{2\pi} \sqrt{\frac{M(B + B_H)}{I}}$

$$\Rightarrow \frac{B}{B_H} = \left(\frac{f'}{f}\right)^2 - 1$$

ILLUSTRATION 3.19

A vibration magnetometer consists of two identical bar magnets placed one over the other, such that they are mutually perpendicular and bisect each other. The time period of oscillation in a horizontal magnetic field is 4 s. If one of the magnets is taken away, find the period of the other in the same field.

Sol. For combination of the two magnets: let M' be the net magnetic moment of the combination of the two magnets and I' , the moment of inertia of the system. If T' is period of vibration, then

$$T' = 2\pi \sqrt{\frac{I'}{M'B}}$$

When two magnets, each of magnetic moment M and moment of inertia I are placed perpendicular to each other, the moment of inertia of the combination becomes

$$I' = 2I$$

and the magnetic moment of the combination is given by

$$M' = \sqrt{M^2 + M^2 + 2MM \cos 90^\circ} = \sqrt{2} M$$

Since $T' = 4$ s, we have

$$\therefore 4 = 2\pi \sqrt{\frac{2I}{\sqrt{2}MB}} \quad \dots(i)$$

When one magnet is taken away:

$$T = 2\pi \sqrt{\frac{I}{MB}} \quad \dots(ii)$$

Dividing Eq. (ii) by (i), we have

$$\frac{T}{4} = 2\pi \sqrt{\frac{I}{MB}} / 2\pi \sqrt{\frac{2I}{\sqrt{2}MB}}$$

or $T = 4 \times (2)^{1/4} = 4 \times 1.189 = 4.76$ s

CONCEPT APPLICATION EXERCISE 3.2

1. The horizontal component of earth's magnetic field at a place is $\sqrt{3}$ times the vertical component. What is the value of angle of dip at this place?
2. The horizontal component of earth's magnetic field at a place is 0.3×10^{-4} T. If the angle of dip is 60° , what is the total earth's magnetic field?
3. At 52° from the magnetic meridian, a magnetic needle in a vertical plane makes an angle of 45° with the horizontal plane. Find the actual angle of dip at that place. [$\cos 52^\circ = 0.6157$]
4. A small bar magnet has a magnetic moment 5 Am^2 . The neutral point is obtained on axial line, when it is placed in magnetic meridian with its north pole pointing south of earth and neutral point is obtained on equatorial line, when it is placed with its north pole pointing north of earth. If horizontal component of earth's field is 0.38 G , find the position of neutral points in two cases.
5. A short bar magnet is placed in a horizontal plane with its axis in the magnetic meridian. Null points are found on its equatorial line (i.e. its normal bisector) at 12.5 cm from the centre of the magnet. The earth's magnetic field at the place is 0.38 G and the angle of dip is zero.
 - (a) What is the total magnetic field at points on the axis of the magnet located at the same distance (12.5 cm) as the null-points from the centre?
 - (b) Locate the null-points, when the bar is turned around by 180° .

Assume that the length of the magnet is negligible as compared to the distance of the null-points from the centre of the magnet.
6. A bar magnet of length 8 cm and having a pole strength of 1 Am is placed vertically on a horizontal table with its N-pole on the table. A neutral point is found on the table at a distance of 6 cm south of the magnet. Calculate the earth's horizontal magnetic field.

ANSWERS

1. 30° 2. $0.6 \times 10^{-4} \text{ T}$ 3. $\tan^{-1} 0.6157$
 4. 29.7 cm, 23.6 cm
 5. (a) 144 G; (b) 15.7 cm 6. $2.18 \times 10^{-5} \text{ T}$

MAGNETIC PROPERTIES OF MATERIALS

The magnetic properties of a material are described in terms of following physical parameters. These properties are used to classify the magnetic substances and to understand their response to the magnetic fields:

INTENSITY OF MAGNETIZATION

It is defined as the magnetic moment developed per unit volume, when a magnetic specimen is subjected to magnetizing field. It is denoted by I .

If a magnetic specimen of volume V acquires magnetic dipole moment M due to the magnetizing field, then

$$I = \frac{M}{V} \quad \dots(i)$$

Since unit of magnetic moment is ampere metre², the unit of intensity of magnetization is

$$[I] = \frac{\text{ampere metre}^2}{\text{metre}^3} = \text{ampere metre}^{-1} (\text{Am}^{-1})$$

i.e. intensity of magnetization and the magnetic intensity have the same units. The intensity of magnetization is sometimes called simply magnetization.

If the magnetic specimen has area of cross-section a and length $2l$, then its volume,

$$V = a \times 2l$$

Further, if m is pole strength developed, then magnetic moment of the specimen,

$$M = m \times 2l$$

Therefore, Eq. (i) gives

$$I = \frac{m \times 2l}{a \times 2l} = \frac{m}{a} \quad \dots(ii)$$

Hence, intensity of magnetization may also be defined as the pole strength developed per unit area of cross-section of the specimen.

Relation Between Intensity of Magnetization and Magnetic Field or Induction of an Ideal Solenoid

If a solenoid with n turns per unit length is carrying a current i , magnetic moment of the solenoid due to each turn of the solenoid $= i_M A$.

Total magnetic moment of the material due to all the turns (N) of the solenoid $= Ni_M A$

Here, A is the area of cross-section of the solenoid and l is its length.

Clearly, $M = \frac{\text{magnetic moment of the material}}{\text{volume of the material (i.e., solenoid)}}$

$$\text{or } M = \frac{Ni_M A}{Al} = ni_M \quad (\text{here } N/l = n)$$

If B_M is the additional magnetic induction (or field) set up by the material of the core.

$$\text{Then, } B_M = \mu_0 ni_M$$

$$\text{Hence, } B_M = \mu_0 M \quad \dots(iii)$$

MAGNETIC INTENSITY

Consider that inside vacuum, a magnetic field B_0 exists. Then, magnetic intensity is given by the relation

$$H = \frac{B_0}{\mu_0}, \quad \dots(iv)$$

where $\mu_0 = 4\pi \times 10^{-7} \text{ tesla metre ampere}^{-1}$ is absolute permeability of vacuum. Since unit of magnetic field is tesla, the unit of magnetic intensity is

$$[H] = \frac{\text{tesla}}{\text{tesla metre ampere}^{-1}} = \text{ampere metre}^{-1} (\text{Am}^{-1})$$

Magnetic intensity is also known as H -field or magnetic field strength. The unit of magnetic intensity i.e. Am^{-1} is also equivalent to $\text{N m}^{-2} \text{ T}^{-1}$ or N Wb^{-1} or $\text{J m}^{-3} \text{ T}^{-1}$ or $\text{J m}^{-1} \text{ Wb}^{-1}$.

The older unit of magnetic intensity is oersted. It is related to gauss (the older unit of magnetic field) as given below:

$$1 \text{ oersted} = \frac{1 \text{ gauss}}{\mu_0}$$

$$\text{Therefore, } 1 \text{ oersted} = \frac{10^{-4} \text{ T}}{4\pi \times 10^{-7} \text{ T m A}^{-1}} \approx 80 \text{ Am}^{-1}$$

Important Point:

A solenoid with n turns per unit length is carrying a current i . Clearly, magnetic field (or induction) inside the empty solenoid, i.e., $B_0 = \mu_0 ni$.

Then magnetic intensity is given by the relation

$$H = \frac{B_0}{\mu_0} = \frac{\mu_0 ni}{\mu_0} = ni \quad \dots(v)$$

Thus product of n and i , i.e., ni is called the magnetizing field intensity, H .

TOTAL MAGNETIC FIELD OR INDUCTION

Consider that a soft iron bar (ferromagnetic material) is placed in a uniform magnetic field B_0 set up in vacuum. Let us consider a solenoid with n turns per unit length, carrying a current i . Clearly, magnetic field (or induction) inside the empty solenoid, i.e.,

$$B_0 = \mu_0 ni_0 = \mu_0 H \quad \dots(vi)$$

If we fill the solenoid by a magnetic material as a core, the total or resultant magnetic field, i.e.,

$$B = B_0 + B_M \quad \dots(vii)$$

Here, B_M is the additional magnetic induction (or field) set up by the material of the core. Further,

$$B_M = \mu_0 ni_M = \mu_0 M \quad \dots(viii)$$

where i_M is the magnetization current (or Amperean current) arising on account of the magnetization of the material.

It is to be noted that B_M is produced by small circulating currents inside the magnetic material due to orbiting and spinning electrons in the atoms.

From Eqs. (vi), (vii) and (viii), we get

$$B = \mu_0 ni + \mu_0 ni_M = \mu_0 (H + M) \quad \dots(\text{ix})$$

The magnetic induction is defined as the number of magnetic lines of induction (magnetic field lines inside the material) crossing per unit area normally through the magnetic substance. It is denoted by B .

Magnetic induction is also known as magnetic flux density or simply magnetic field. The SI unit of magnetic induction is tesla (T) or weber metre⁻² (Wb m⁻²). These units of magnetic induction are equivalent to N m⁻¹ A⁻¹ or J A⁻¹ m⁻².

The older unit of magnetic induction is **gauss (G)**.

$$1 \text{ tesla} = 10^4 \text{ gauss (G)}$$

Important Point:

Equation (ix) can be written as $H = \frac{B}{\mu_0} - M$

Since $\vec{B}$ and $\vec{M}$ are vectors, $\vec{H}$ is also a vector. Hence, the magnetic intensity vector.

$$\text{Thus, } \vec{H} = \frac{\vec{B}}{\mu_0} - \vec{M}$$

MAGNETIC SUSCEPTIBILITY (χ_m)

For a large class of isotropic materials, $M \propto H$

$$\text{or } M = \chi_m H \quad \dots(\text{x})$$

where χ_m is a dimensionless quantity and is called the magnetic susceptibility.

MAGNETIC PERMEABILITY (μ)

Total magnetic field or induction can be given by the relation,

$$B = \mu_0 (H + M)$$

Hence we can write, $B = \mu_0 (H + \chi_m H) = \mu_0 (1 + \chi_m) H$

$$\text{or } B = \mu H \quad \dots(\text{xi})$$

The constant μ is called the absolute magnetic permeability, where

$$\mu = \mu_0 (1 + \chi_m) \quad \dots(\text{xii})$$

Further, $\mu/\mu_0 = \mu_r$ is called the relative permeability of the material.

$$\text{From Eq. (xii), } \mu_r = 1 + \chi_m \quad \dots(\text{xiii})$$

Important Points:

- $\frac{B}{B_0} = \frac{\mu H}{\mu_0 H} = \frac{\mu}{\mu_0} = \mu_r$
- The unit of μ is the same as that of μ_0 , i.e., Wb/Am. μ_r is a pure ratio and has no unit.

MAGNETIC FLUX

Magnetic flux through a surface is defined as the number of magnetic field lines passing normally through the surface. It is denoted by (ϕ).

Consider that a surface area $\Delta \vec{S}$ is held in a magnetic field of induction B . Then, magnetic flux passing normally through the surface is given by

$$\phi = \vec{B} \cdot \Delta \vec{S}$$

The SI unit of magnetic flux is weber (Wb).

ILLUSTRATION 3.20

The absolute magnetic permeability is measured to be $0.12 \text{ T A}^{-1} \text{ m}$. Find its relative permeability and susceptibility.

Sol. Here $\mu = 0.12 \text{ T A}^{-1} \text{ m}$

We know $\mu_0 = 4\pi \times 10^{-7} \text{ T A}^{-1} \text{ m}$

$$\therefore \mu_r = \frac{\mu}{\mu_0} = \frac{0.12}{4\pi \times 10^{-7}} = 9.55 \times 10^4$$

$$\text{Also, } \chi_m = \mu_r - 1 = 9.55 \times 10^4 - 1 = 9.55 \times 10^4$$

ILLUSTRATION 3.21

A magnetizing field of 1500 A/m produces a magnetic flux of $2.4 \times 10^{-5} \text{ Wb}$ in a bar of iron of cross-section 0.5 cm^2 . Calculate permeability and susceptibility of iron bar used.

Sol. Here, $H = 1500 \text{ A/m}$, $\phi_B = 2.4 \times 10^{-5} \text{ Wb}$

$$A = 0.5 \text{ cm}^2 = 0.5 \times 10^{-4} \text{ m}^2$$

$$\begin{aligned} \text{Magnetic field, } B &= \frac{\phi_B}{A} = \frac{2.4 \times 10^{-5} \text{ Wb}}{0.5 \times 10^{-4} \text{ m}^2} \\ &= 0.48 \text{ Wb/m}^2 \end{aligned}$$

$$\text{Permeability, } \mu = \frac{B}{H} = \frac{0.48 \text{ Wb/m}^2}{1500 \text{ A/m}} = 3.2 \times 10^{-4} \frac{\text{Tm}}{\text{A}}$$

$$\begin{aligned} \text{Susceptibility, } \chi_m &= \frac{\mu}{\mu_0} - 1 = \left(\frac{3.2 \times 10^{-4}}{4\pi \times 10^{-7}} \right) - 1 \\ &= 255 - 1 = 254 \end{aligned}$$

CLASSIFICATION OF MAGNETIC MATERIALS

On the basis of mutual interactions or behavior of various materials in an external magnetic field, the materials are classified into three major categories, namely (a) diamagnetic (b) paramagnetic and (c) ferromagnetic

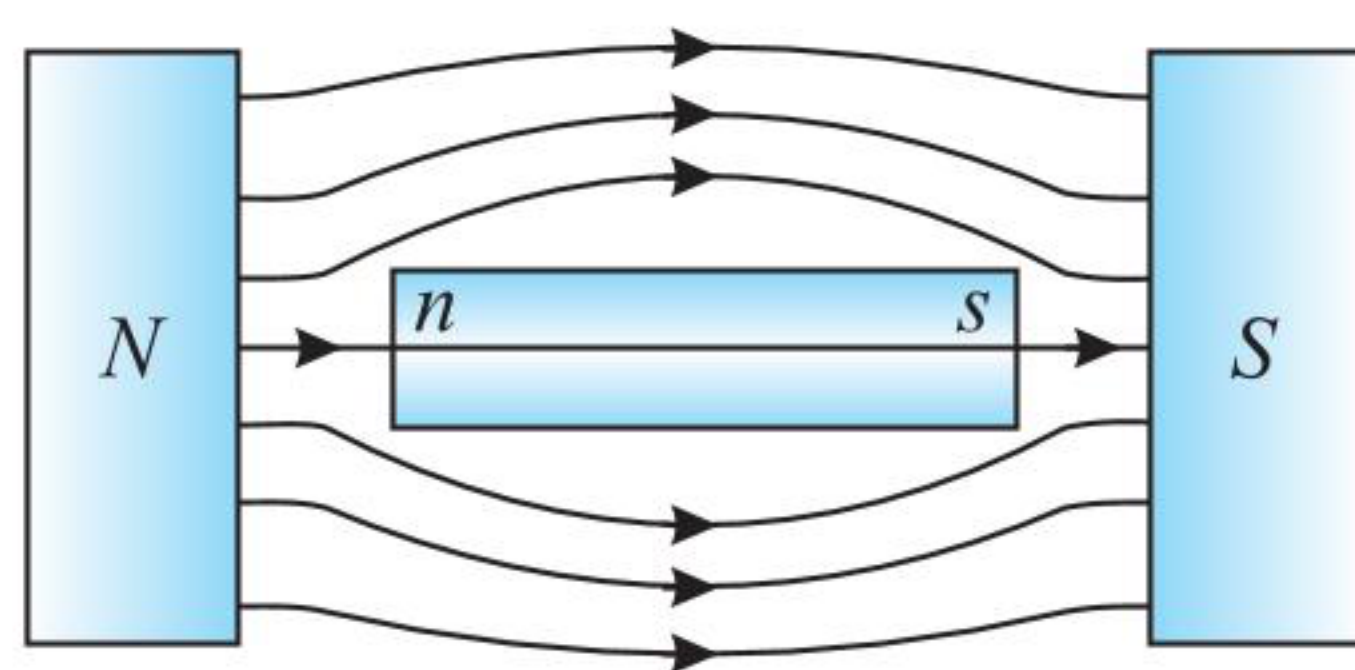
DIAMAGNETIC SUBSTANCES

The substances which are feebly magnetized in a direction opposite to that of magnetizing field in which these are placed are called diamagnetic substances.

For example; bismuth, antimony, copper, gold, quartz, mercury, water, alcohol, air, hydrogen, etc., are all diamagnetic substances. Some other properties of diamagnetic substances are:

- When a diamagnetic substance is placed inside an external magnetic field, the magnetic field inside the diamagnetic is found to be slightly less than the external magnetic field.

- When a diamagnetic material is placed inside a magnetic field, the magnetic field lines become slightly less dense in the diamagnetic material



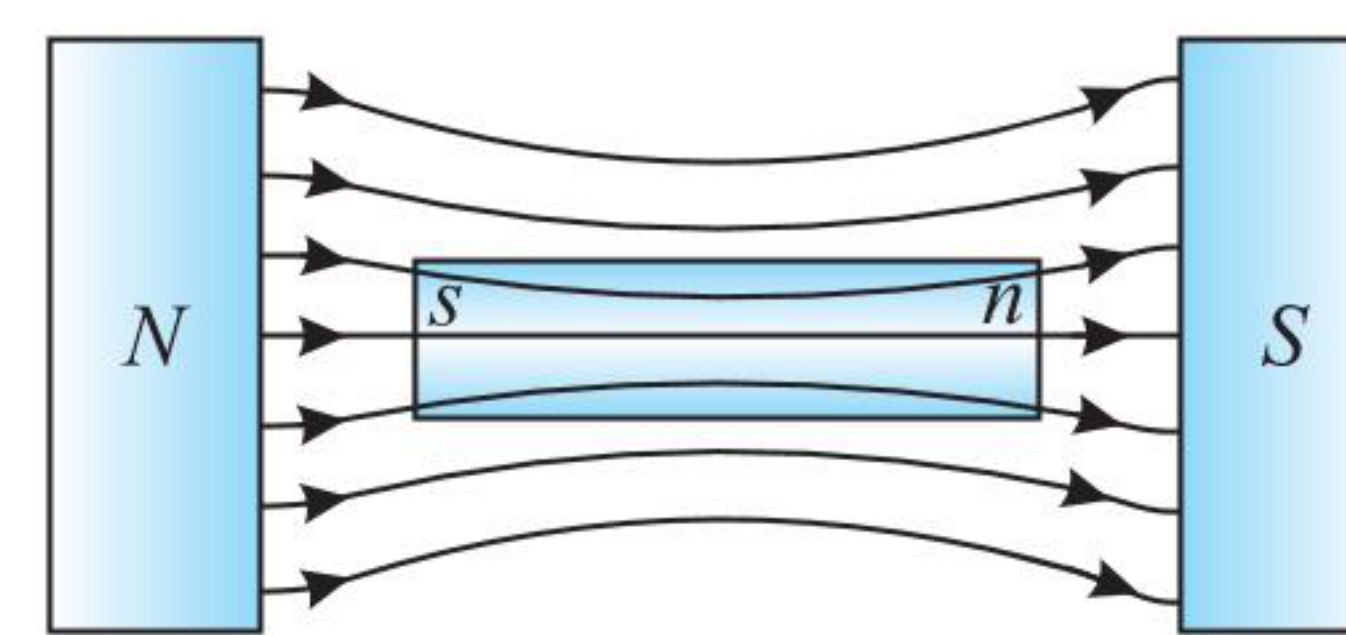
- When a diamagnetic substance is placed in a magnetic field, a small magnetization occurs opposite to the direction of the field. As a result, the magnetic field lines become less dense inside the diamagnetic material.
- It is observed that when a diamagnetic sample is placed inside a nonuniform magnetic field, it tends to move from stronger part to the weaker part of the magnetic field. It may be pointed out that the diamagnetic effects are too feeble to be detected, unless the applied magnetic field is strong.
- The behavior of a diamagnetic substance is independent of temperature.
- A diamagnetic substance has the nature similar to that of a dielectric having non-polar atoms.
- For a diamagnetic substance, the intensity of magnetization (I) has a small negative value. It is because, on being placed in a magnetic field, a diamagnetic substance gets slightly magnetized in a direction opposite to that of the applied field.
- The relative permeability of diamagnetic substance is less than unity.
- As $\mu_r = 1 + \chi_m$ and $\mu_r < 1$, χ_m is negative, i.e., the susceptibility of diamagnetic is negative.
- Susceptibility of diamagnetic does not change with temperature. *Bi* at low temperatures is an exception to this general property.

PARAMAGNETIC SUBSTANCES

The substances which are feebly magnetized in the direction of the magnetizing field in which these are placed are called paramagnetic substances.

For example, aluminum, platinum, chromium, manganese, copper sulphate, crown glass, solutions of salts of iron and nickel and oxygen are all paramagnetic substances. Some of the other properties of paramagnetic substance are:

- When a paramagnetic substance is placed inside an external magnetic field, the magnetic field inside the paramagnetic is found to be a slightly greater than the external magnetic field. In contrast to the behavior of diamagnetic,
- A paramagnetic substance tends to move from weaker part of the magnetic field to stronger part, when placed in a non-uniform magnetic field.
- In contrast to diamagnetic, the behavior of a paramagnetic is temperature dependent. Also, the paramagnetic effects are perceptible only with a strong magnetic field. The nature of a paramagnetic is similar to that of a dielectric having polar atoms.
- When a paramagnetic material is placed inside a magnetic field, the magnetic field lines become slightly more dense in the paramagnetic material



- When a paramagnetic substance is placed in a magnetic field, a small magnetization occurs in the direction of the field. As a result, the magnetic field lines become more dense inside the paramagnetic substance.
- The relative permeability of paramagnetic substances is greater than unity.
- As $\mu_r = 1 + \chi_m$ and $\mu_r > 1$, χ_m is positive. Hence, the susceptibility of paramagnetics is positive.
- Susceptibility of paramagnetic varies inversely as the temperature of the substance, i.e.,

$$\chi_m \propto 1/T \text{ or } \chi_m = C/T \quad \dots(i)$$

where C = Curie constant depending upon the nature of the substance and T = absolute temperature.

Equation (i) is called the **Curie's law** which was experimentally established by the French physicist and chemist, Pierre Curie.

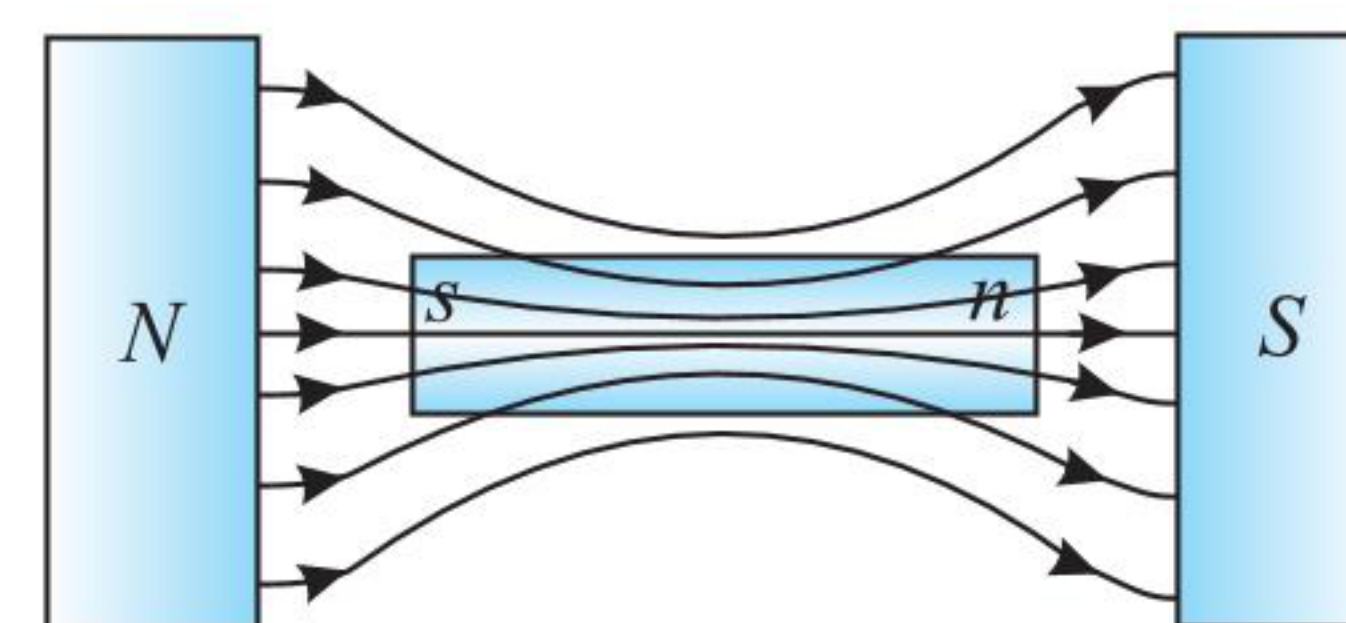
FERROMAGNETIC SUBSTANCES

The substances which are strongly magnetized in the direction of the magnetizing field in which they are placed are called ferromagnetic substances.

For example, iron, cobalt, nickel, gadolinium and a number of alloys (Alnico, Nipennag) are ferromagnetic in nature.

The ferromagnetic materials exhibit the properties of paramagnetic substances to a much higher degree. For example:

- When a ferromagnetic substance is placed inside a magnetic field, the field inside the ferromagnetic substance gets greatly enhanced. As a result, when a ferromagnetic is placed in a non-uniform magnetic field, it quickly moves from weaker part to stronger part of the magnetic field. In other words, the ferromagnetic effects are perceptible even in the presence of a weak magnetic field.
- When a ferromagnetic material is placed inside a magnetic field, the magnetic field lines becomes highly dense in the ferromagnetic substance as shown in figure.



- When a ferromagnetic material is placed in a magnetic field, a large magnetization occurs in the direction of magnetic field.
- For a ferromagnetic substance, the intensity of magnetization (I) has a large positive value. It is because, on being placed in a magnetic field, a ferromagnetic substance gets strongly magnetized in the direction of the field.
- Permeability of ferromagnetic materials is very large, of the order of hundreds and thousands.

- Similarly, their susceptibility is also very large. That is why these can be magnetized easily and strongly.
- The ferromagnetic behavior of a substance becomes temperature dependent. With the rise of temperature, susceptibility of ferromagnetic materials decreases. At a certain temperature, ferromagnetics pass over to paramagnetics. This transition temperature is called **Curie temperature or Curie point (T_c)**. For example, Curie temperature for iron is 770°C and 365°C for nickel and only 70°C for permalloy (an alloy of 70% Fe and 30% Ni).
- At a temperature above the Curie point, a ferromagnetic becomes an ordinary paramagnetic whose magnetic susceptibility obeys the **Curie-Weiss law**.

According to Curie-Weiss law, at temperatures above Curie temperature the magnetic susceptibility of ferromagnetic materials is inversely proportional to $(T - T_c)$

$$\text{i.e., } \chi \propto \frac{1}{T - T_c} \Rightarrow \chi = \frac{C}{(T - T_c)}$$

Here T_c = Curie temperature.

χ - T curve is shown (for Curie-Weiss Law).

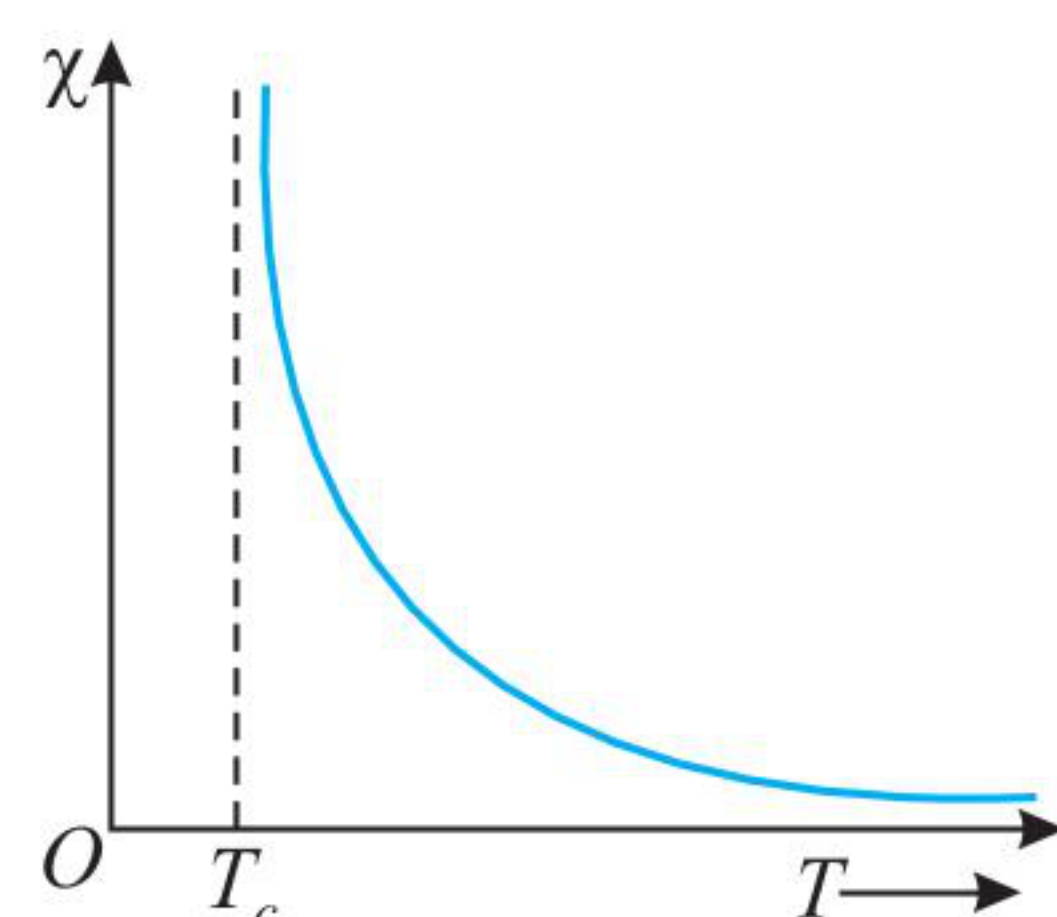


ILLUSTRATION 3.22

For a magnetizing field of intensity $2 \times 10^3 \text{ Am}^{-1}$, aluminum at 280 K acquires intensity of magnetization of $4.8 \times 10^{-2} \text{ Am}^{-1}$. Find the susceptibility of aluminum at 280 K. If the temperature of the aluminum is raised to 320 K, what will be its susceptibility and intensity of magnetization?

Sol. Here, $H = 2 \times 10^3 \text{ Am}^{-1}$;

$$I = 4.8 \times 10^{-2} \text{ Am}^{-1} \text{ and } T = 280 \text{ K}$$

$$\text{Now, } \chi_m = \frac{I}{H} = \frac{4.8 \times 10^{-2}}{2 \times 10^3} = 2.4 \times 10^{-5}$$

Aluminum is a paramagnetic substance and hence obeys Curies' law. If χ_m' is susceptibility of Aluminum at temperature $T' (= 320 \text{ K})$, then

$$\frac{\chi_m'}{\chi_m} = \frac{T}{T'} \text{ or } \chi_m' = \chi_m \times \frac{T}{T'}$$

$$\text{or } \chi_m' = 2.4 \times 10^{-5} \times \frac{280}{320} = 2.1 \times 10^{-5}$$

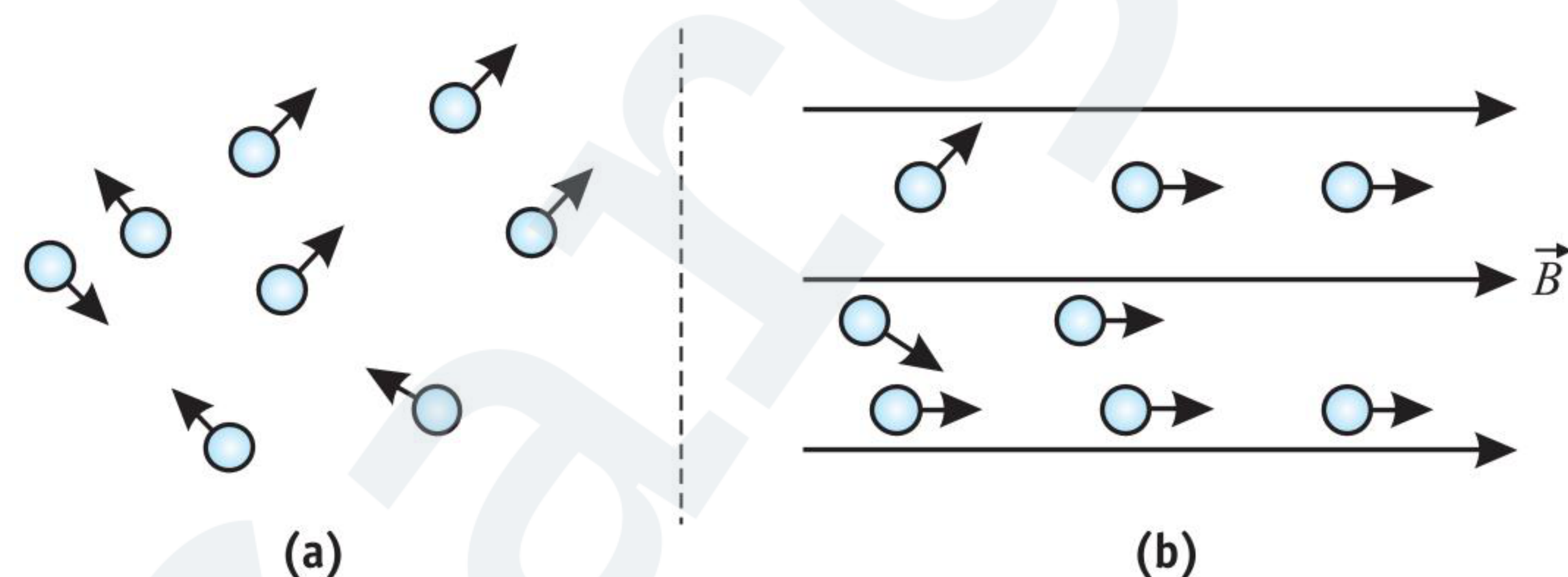
If I' is intensity of magnetization of aluminum at 320 K, then

$$I' = \chi_m' H = 2.1 \times 10^{-5} \times 2 \times 10^3 = 4.2 \times 10^{-2} \text{ Am}^{-1}$$

PARAMAGNETISM ON BASIS OF ELECTRON THEORY

An atom of a paramagnetic material possesses resultant magnetic moment as the magnetic moments of the electrons in such an

atom do not exactly cancel. Each atom, therefore, behaves like a tiny magnet. In the absence of any external magnetic field, the resultant magnetic moment due to all the atoms is zero as these moments are randomly oriented as shown in Fig. (a). When an external magnetic field is applied to a paramagnetic material, the magnetic moments of its atoms tend to align with the direction of the magnetic field, giving rise to a small net magnetic moment. However, thermal motion interferes with the aligning process and as such does not permit a complete alignment as shown in Fig. (b).



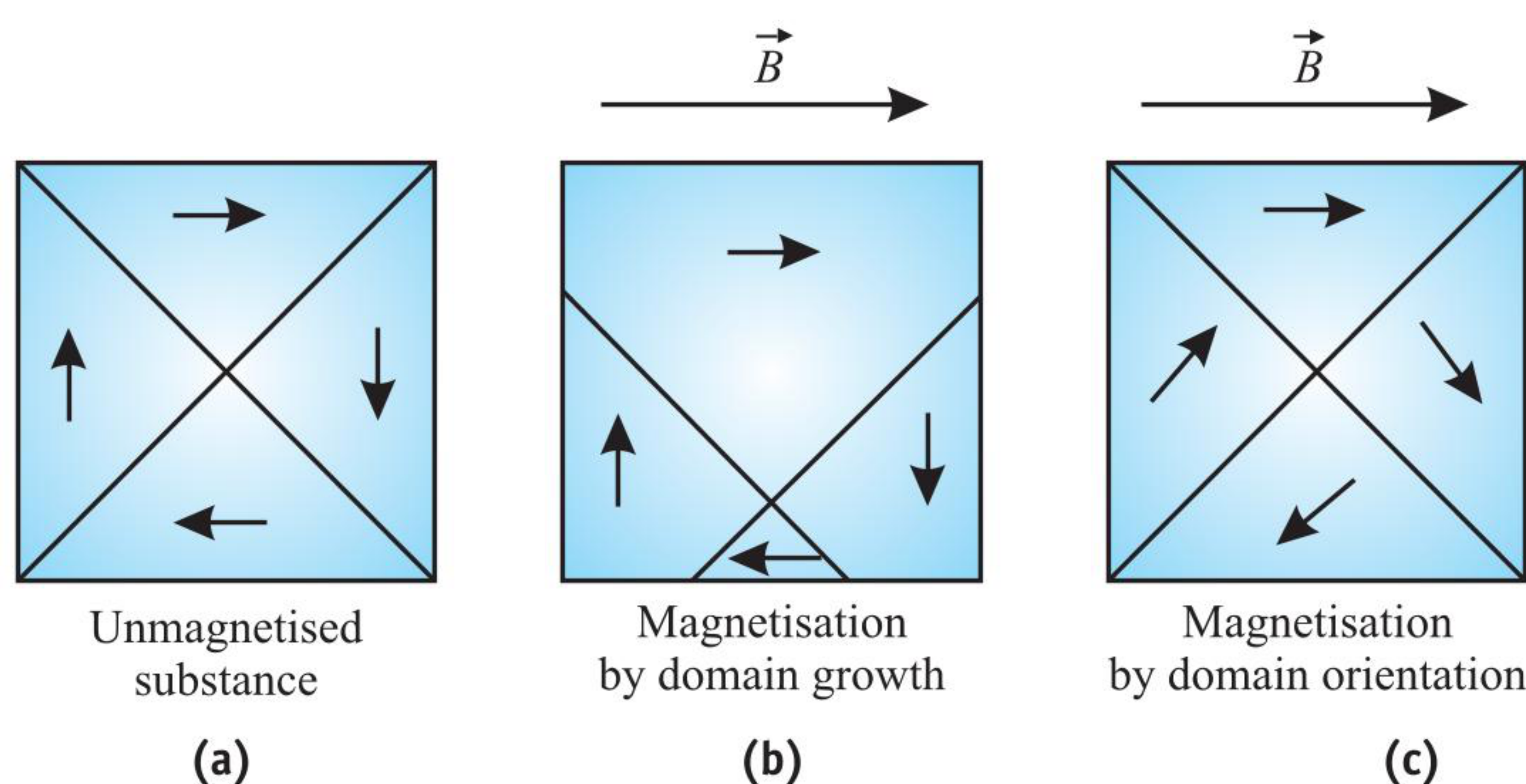
Obviously, the paramagnetic behavior decreases with rise in temperature. On the contrary, when a paramagnetic salt is cooled, the magnetic moments of the atoms of the salt tend to align with the magnetic field intensity, thereby increasing its magnetization.

The magnetic susceptibility of paramagnetic materials is about a hundred times higher than that for diamagnetic substances.

FERROMAGNETISM

Ferromagnetic materials possess a very large resultant magnetic moment. The fundamentals of the theory of ferro-magnetism were given by Y. Frenkel and Werner Heisenberg. This theory is called the **domain theory**. According to this theory:

- The spin magnetic moments of the electrons are responsible for the magnetic properties of ferromagnetics.
- Under certain definite forces, these spin magnetic moments are lined up parallel to one another. This results in setting up of regions of spontaneous magnetization, which are called **domains**.
- In each domain, the atoms have magnetic moments which are aligned strictly in a single direction.
- A very strong magnetic field appears inside a domain and as a result of this, it is magnetised to saturation. A domain contains from 10^{21} to 10^{17} atoms and has a dimension of the order of 10^{-8} to 10^{-12} m^3 .
- Magnetic moments of different domains have different directions in the absence of external magnetic field and as a result of this, the resultant magnetic moment is zero as shown in Fig. (a).
- In the presence of an external magnetic field ($\vec{B}$), the domains suffer two effects: (a) those domains oriented favourably with respect to the magnetic field grow at the expense of those oriented less favorably [Fig. (b)] due to reorientation effect of the magnetic field (b) the magnetisation of the domains tends to align in the direction of the field [Fig. (c)] and the piece of matter becomes a magnet.

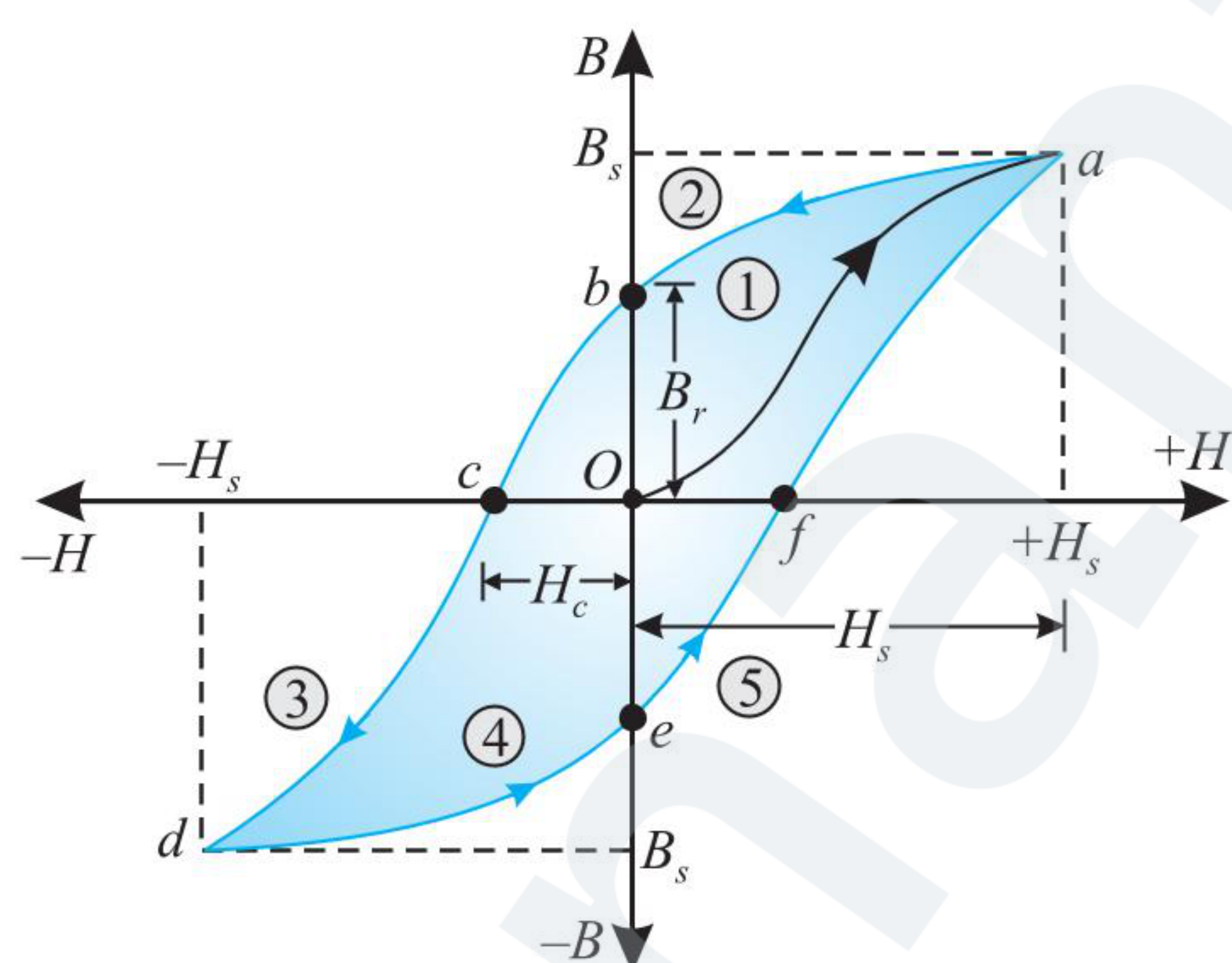


HYSTERESIS

A specimen of ferromagnetic material is placed in a magnetizing field whose strength and direction can be changed. Suppose that the specimen is unmagnetized initially. When the magnetizing field (H) is increased, the intensity of magnetization (I) of the material of the specimen also increases. It is found that when the magnetizing field is made zero, the intensity of magnetization does not become zero but still has some finite value. It becomes zero only, when magnetizing field is increased in reverse direction. In other words, intensity of magnetization does not become zero on making magnetizing field zero but does so a little late and this effect is called hysteresis.

Hysteresis is the phenomenon shown by ferromagnetic substances where by the magnetic field induction (B) of the medium depends not only on the existing magnetising field (H) but also on the previous state of the substance.

The relationship between B and H in a ferromagnetic substance is not linear and is quite complex. In order to study hysteresis, let us take a ferromagnetic substance through a cycle of magnetization.



- When H changes from 0 to $+H_s$, B changes along Oa (path 1)
- When H changes from $+H_s$ to 0, B changes along ab (path 2)
- When H changes from 0 to $-H_s$, B changes along bed (path 3)
- When H changes from $-H_s$ to 0, B changes along de (path 4)
- When H changes from 0 to $+H_s$, B changes along efa (path 5)

The graph $abcdefa$ between B and H is called the hysteresis loop or the hysteresis curve as shown in figure.

SALIENT FEATURES OF HYSTERESIS LOOP

- When $H = 0$ (after having increased from 0 to H_s), $B_r = B$. Here, B_r is B called the residual magnetism, retentivity or remanence. Thus, retentivity is the property by virtue of which the magnetism (I) remains in a material even on the removal of magnetizing field.
- When magnetic field H is reversed, magnetization decreases and for a particular value of H . When $H = H_c$, $B = 0$. This value of H is called the coercivity of the specimen. Thus, coercivity of a material is the measure of the magnetizing field (H) required to destroy the residual magnetism.

Magnetic hard substance (steel) $\rightarrow$ High coercivity

Magnetic soft substance (soft iron) $\rightarrow$ Low coercivity

- The process of taking a ferromagnetic through a cycle of magnetization results in loss of energy. This is called hysteresis loss and it appears in the form of heat.
- Area of the B - H loop is equal to the energy loss per cycle per unit volume since area of B - H loop $= B \times H$

$$= T \times (A/m) \left(\frac{N}{Am} \right) \left(\frac{A}{m} \right) = \frac{Nm}{m^3} = J/m^3.$$

- The maximum value of B is called the saturation field (B_s) and $B_s \sim 10^3 B_0$, where $B_0 = \mu_0 nI$. The value of H corresponding to B_s is the saturation value of magnetising field (H_s).
- A similar curve is obtained if we plot a graph between M and H .
- Unit of area of M - H loop = unit of $M \times H$

$$= \left(\frac{A}{m} \right) \left(\frac{A}{m} \right) = \frac{A^2}{m^2}$$

Further, $\mu_0 \times$ area of M - H loop

$$= \mu_0 \times \frac{A^2}{m^2} = \left(\frac{Wb}{Am} \right) \frac{A^2}{m^2} = \frac{TA}{m}$$

$$= \text{area of } B\text{-}H \text{ loop.}$$

Area of B - H loop $= \mu_0 \times$ area of M - H loop.

- By studying the hysteresis loops of various magnetic materials, we can study the difference in their magnetic properties.

SOFT AND HARD MAGNETIC MATERIALS

The shape of the hysteresis loop is a characteristic of a ferromagnetic substance. It gives the idea about many important magnetic properties of the substance.

- Soft magnetic materials:** They have:

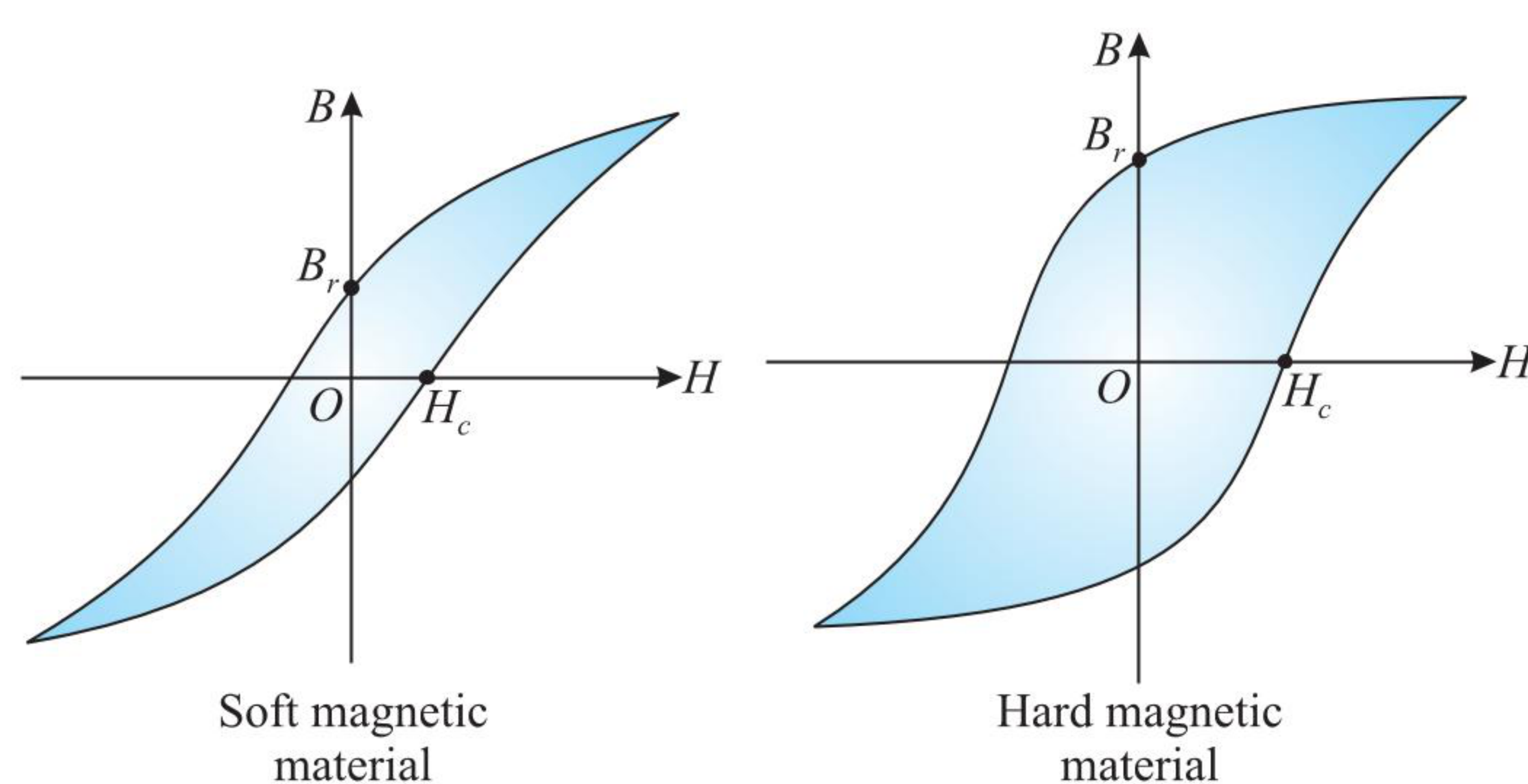
(i) low coercivity (ii) lower retentivity (iii) small hysteresis loss, i.e., small hysteresis loop [Fig. (a)]. These materials are used in:

(a) electromagnets (b) cores of transformers, motors and generators.

- Hard magnetic materials:** They have:

(i) high coercivity (ii) high retentivity (iii) large hysteresis loss, i.e., large hysteresis loop [Fig. (b)]. These materials

are used in making: (a) permanent magnets, (b) magnetic tapes, (c) magnetic core computer memories.



Examples of hard magnetic materials are: Steel, Alnico (Al, Ni, Co), Alcomax, Ticonal (Ti, Co, Ni, Al).

It is caused by charged particles from the Sun that reach the Earth's upper atmosphere and set up currents that change the planet's overall magnetic field.

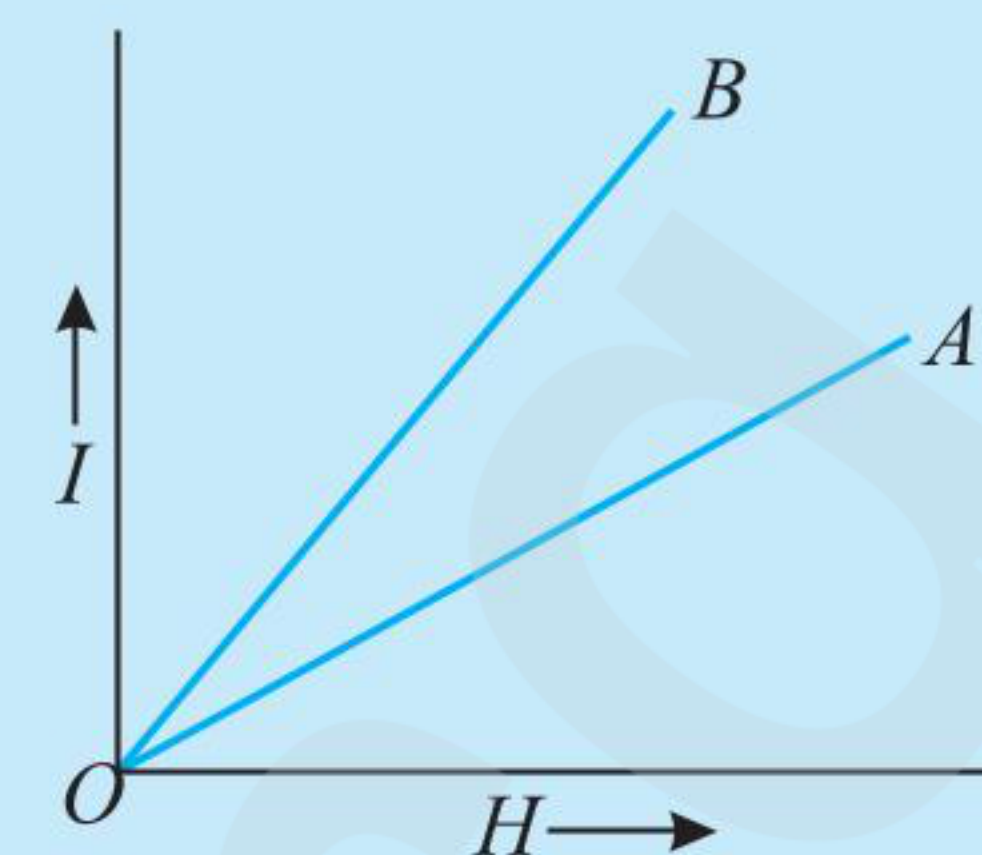
The following conclusions can be drawn from the study of the hysteresis loops of soft iron and steel:

1. The area of hysteresis loop for soft iron is much smaller than that for steel. Therefore, loss of energy per unit volume in case of soft iron will be very small as compared to that in case of steel, when they are taken over a complete cycle of magnetisation and demagnetisation.
2. Soft iron acquires maximum intensity of magnetisation for comparatively much lesser value of magnetising field than in case of steel. In other words, soft iron is much strongly magnetised (or more susceptible to magnetism) than steel.
3. The retentivity of soft iron is greater than that of steel. On removing magnetising field, quite a large amount of magnetisation is retained by soft iron.
4. The coercivity of steel is much larger than that of soft iron. Therefore, the residual magnetism in steel cannot be destroyed that easily as in case of soft iron.

The choice of a particular ferromagnetic material for a practical application is decided from the magnetic properties such as retentivity, coercivity and area of the hysteresis loop.

ILLUSTRATION 3.23

The figure shows the variation of intensity of magnetization (I) versus the applied magnetic field intensity (H) for two magnetic materials A and B .



- (a) Identify the materials A and B .
- (b) Why does the material B has a larger susceptibility than A for a given field at constant temperature?

Sol.

- (a) Paramagnetic materials have a small positive susceptibility (I/H), while ferromagnetic materials have a much larger positive susceptibility. Therefore, material A is paramagnetic and material B is ferromagnetic.
- (b) For a given magnetic field intensity, intensity of magnetisation in case of ferromagnetic materials is very large.

ILLUSTRATION 3.24

A bar magnet has coercivity $4 \times 10^3 \text{ Am}^{-1}$. It is desired to demagnetize it by inserting it inside a solenoid 12 cm long and having 60 turns. What current should be sent through the solenoid?

Sol. The coercivity of $4 \times 10^3 \text{ Am}^{-1}$ of the bar magnet implies that magnetic intensity $H = 4 \times 10^3 \text{ Am}^{-1}$ (in opposite direction) is required to demagnetize it.

Here, number of turns per unit length of the solenoid,

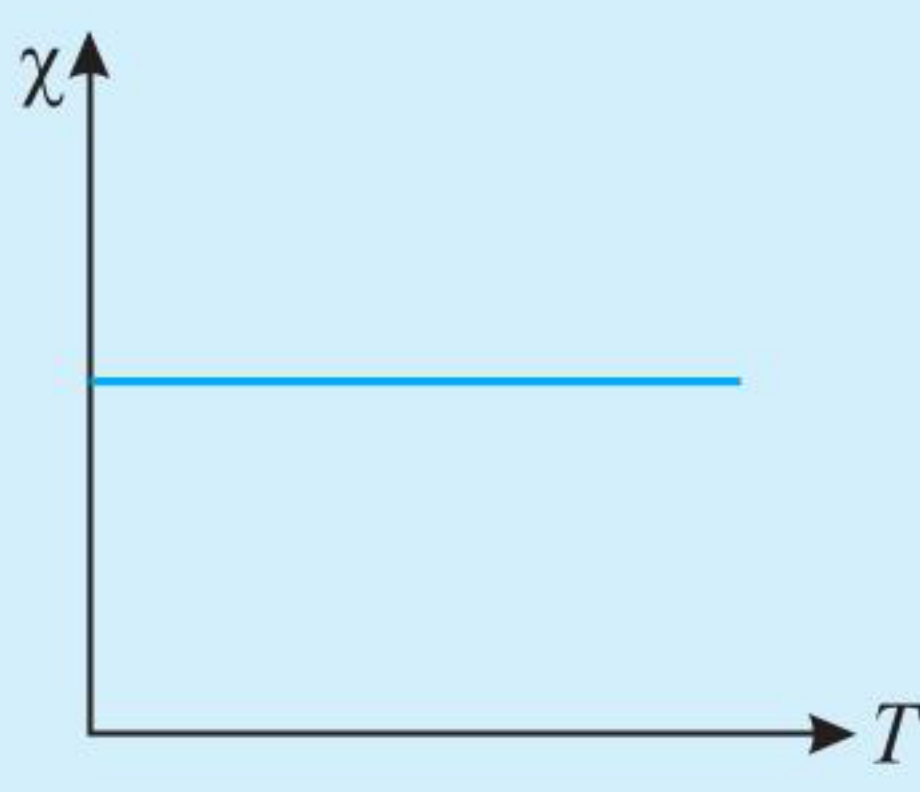
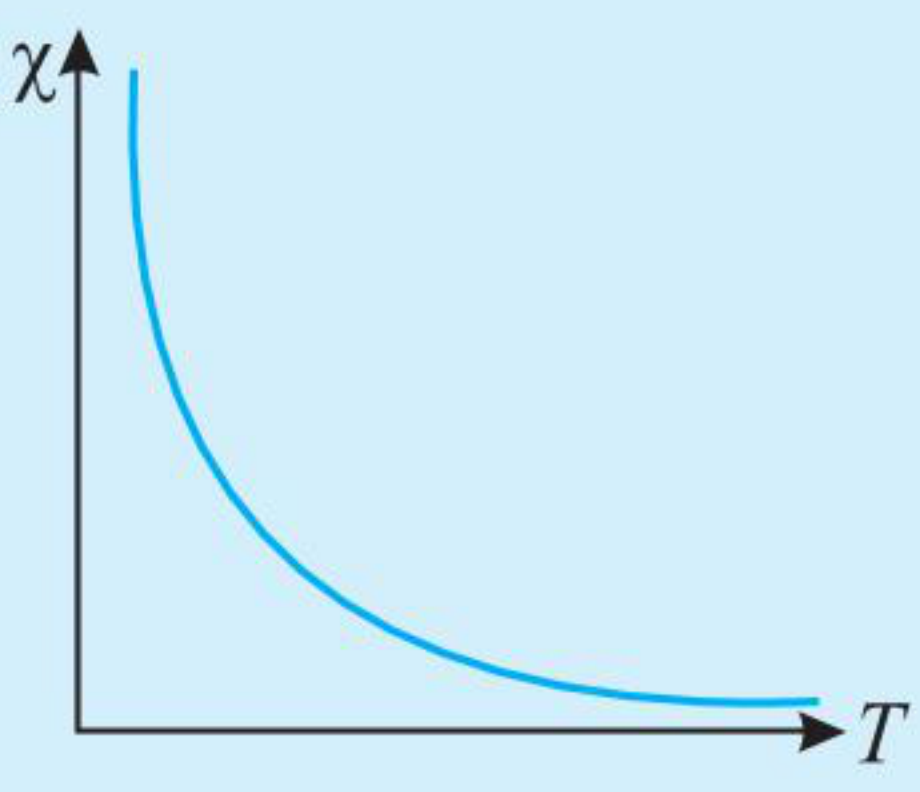
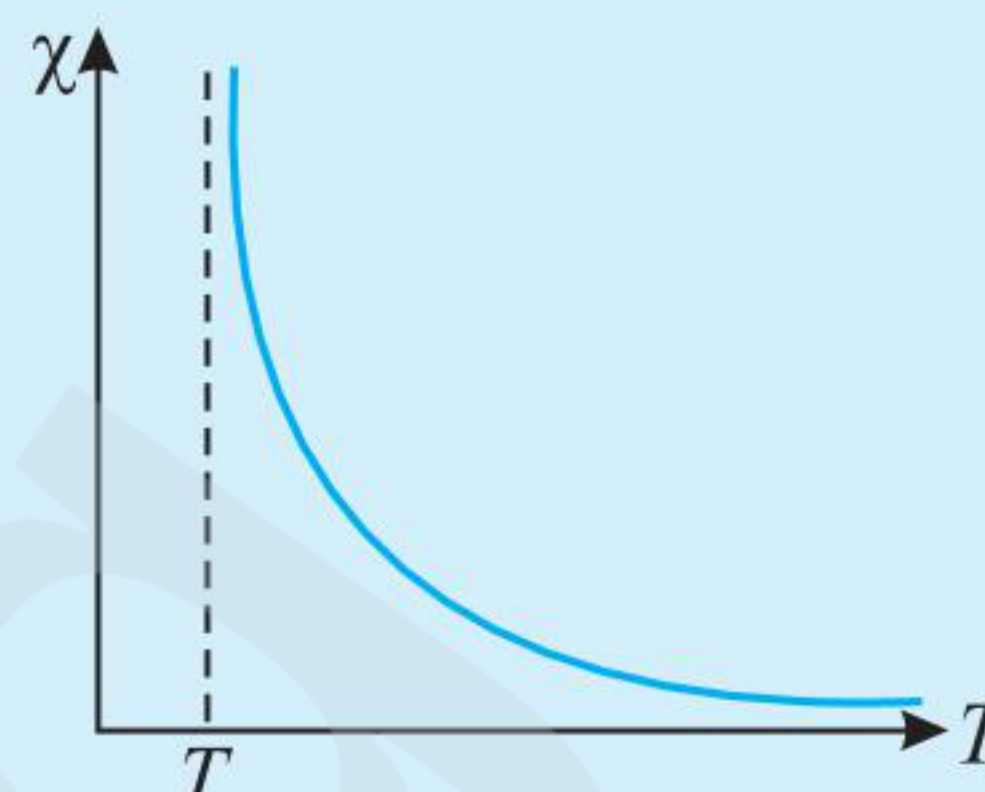
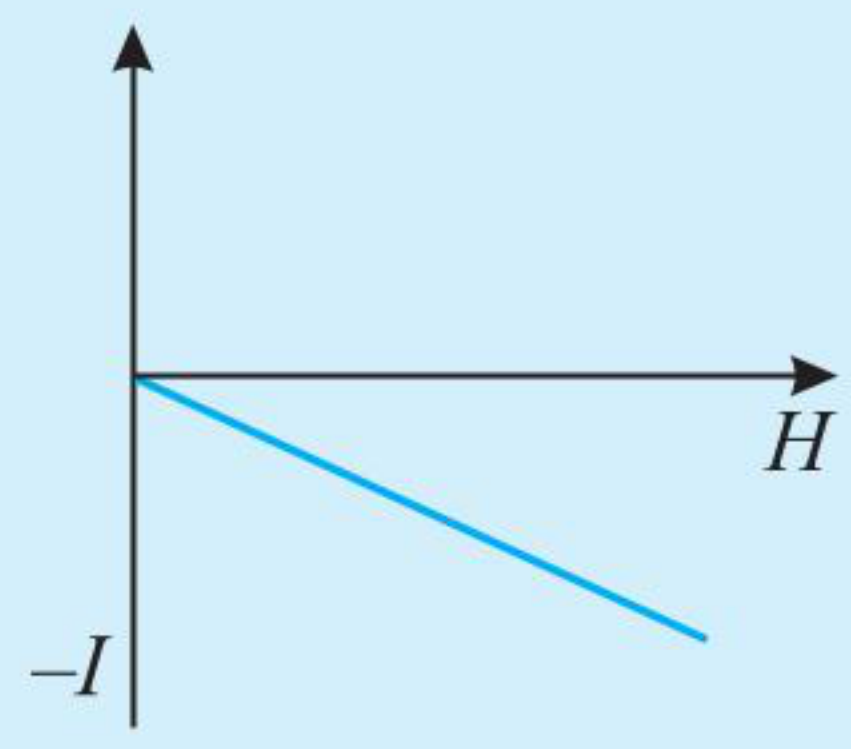
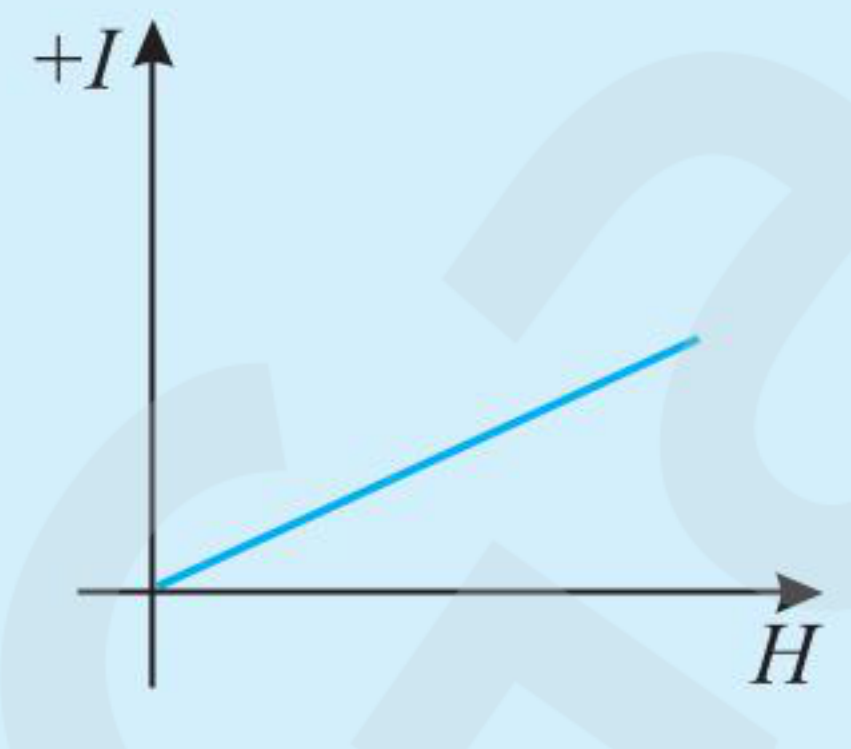
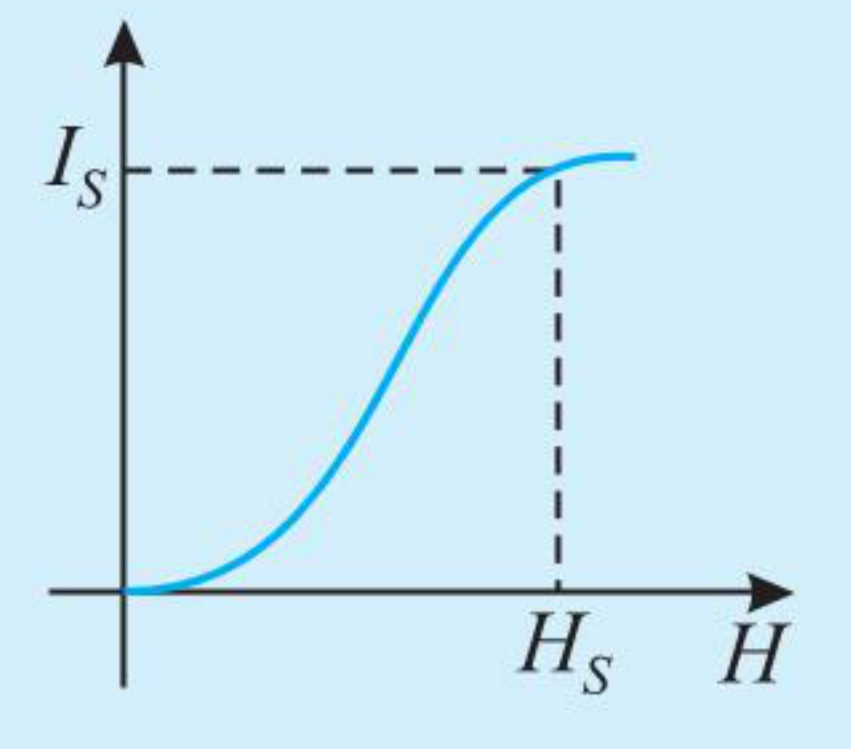
$$n = \frac{60}{12} = 5 \text{ turns cm}^{-1} = 500 \text{ turns m}^{-1}$$

Now, $B = \mu_0 n I$

$$\text{or } I = \frac{B}{\mu_0 n} = \frac{H}{n} = \frac{40 \times 10^3}{500} = 8 \text{ A}$$

Comparative Study of Magnetic Materials

Property	Diamagnetic substances	Paramagnetic substances	Ferromagnetic substances
Cause of magnetism	Orbital motion of electrons	Spin motion of electrons	Formation of domains
Explanation of magnetism	On the basis of orbital motion of electrons	On the basis of spin and orbital motion of electrons	On the basis of domains formed
Value of magnetic induction B	$B < B_0$ (where B_0 is the magnetic induction in vacuum)	$B > B_0$	$B \gg B_0$
Magnetic susceptibility (χ)	Low and negative $ \chi \approx 1$	Low but positive $\chi \approx 1$	Positive and high $\chi \approx 10^2$

Dependence of χ on temperature	Does not depend on temperature (except Bi at low temperature) 	On cooling, these get converted to ferromagnetic materials at Curie temperature 	These get converted into paramagnetic materials at Curie temperature 
Relative permeability (μ_r)	$\mu_r < 1$	$\mu_r > 1$	$\mu_r \gg 1$, $\mu_r = 10^2$
Intensity of magnetisation (I)	I is in a direction opposite to that of H and its value is very low	I is in the direction of H but value is low	I is in the direction of H and value is very high.
I - H curves			
Magnetic moment (M)	Very low (≈ 0)	Very low	Very high
Examples	Cu, Ag, Au, Zn, Bi, Sb, NaCl, H_2O air and diamond etc.	Al, Mn, Pt, Na, $CuCl_2$, O_2 and crown glass	Fe, Co, Ni, Cd, Fe_3O_4 , etc.

CONCEPT APPLICATION EXERCISE 3.3

- A magnet weighs 75 g and its magnetic moment is $2 \times 10^{-4} \text{ Am}^2$. If the density of the material of the magnet is $7.5 \times 10^3 \text{ kg m}^{-3}$, calculate the intensity of magnetization.
- The magnetic induction and magnetizing field in a sample of magnetic material are 1.0 Wb m^{-2} and $2 \times 10^3 \text{ A m}^{-1}$ respectively. Find (a) magnetic permeability (b) relative permeability of the material, (c) magnetic susceptibility and (d) intensity of magnetization. Given that $\mu_0 = 4\pi \times 10^{-7} \text{ T A}^{-1} \text{ m}$.
- The magnetizing field of 1600 A m^{-1} produces a magnetic flux of 2.4×10^{-5} weber in a bar of iron of cross-section 0.2 cm^2 . Calculate relative permeability, intensity of magnetization and susceptibility of the bar.
- The core of a toroid having 3,000 turns has inner and outer radii of 11 cm and 12 cm respectively. The magnetic field in the core for a current of 0.70 A is 2.5 T. What is the relative permeability of the core?
- A Rowland ring has 10^3 turns per unit length. On passing a current of 2 A, magnetic induction is measured to be 1.5 Wb m^{-2} . Calculate (a) relative permeability of the core, (b) magnetic susceptibility, (c) magnetizing field intensity and (d) magnetization.

- The susceptibility of magnesium at 300 K is 1.2×10^{-5} . At what temperature will the susceptibility equal to 144×10^{-5} ?
- The hysteresis loss for a specimen of iron weighing 12 kg is equivalent to $300 \text{ J m}^{-3} \text{ cycle}^{-1}$. Find the loss of energy per hour at 50 cycles s^{-1} . Given, density of iron = $7,500 \text{ kg m}^{-3}$.
- Assume that each iron atom has a permanent magnetic moment equal to 2 Bohr magneton (1 Bohr magneton = $9.27 \times 10^{-24} \text{ A m}^2$). The number density of atoms in iron is $8.52 \times 10^{28} \text{ m}^{-3}$. Find the maximum value of (a) intensity of magnetization and (b) magnetic induction in an iron bar.

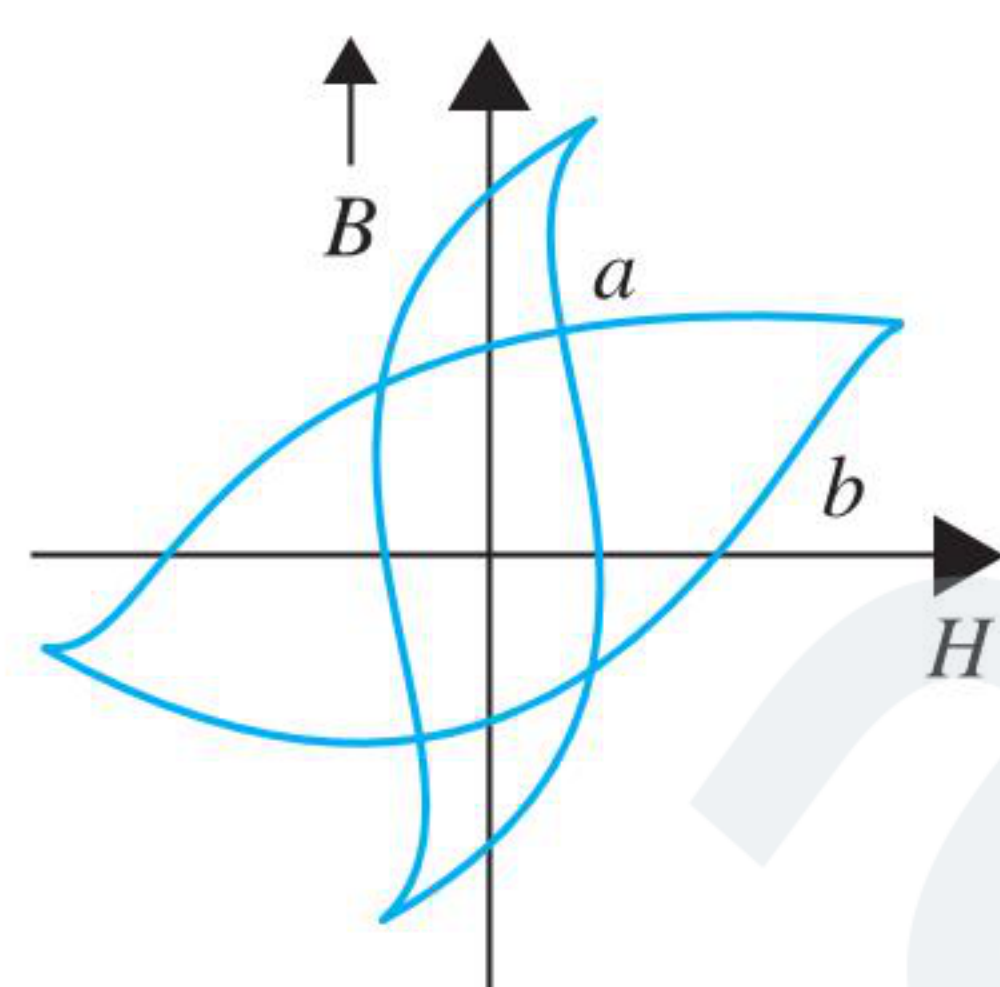
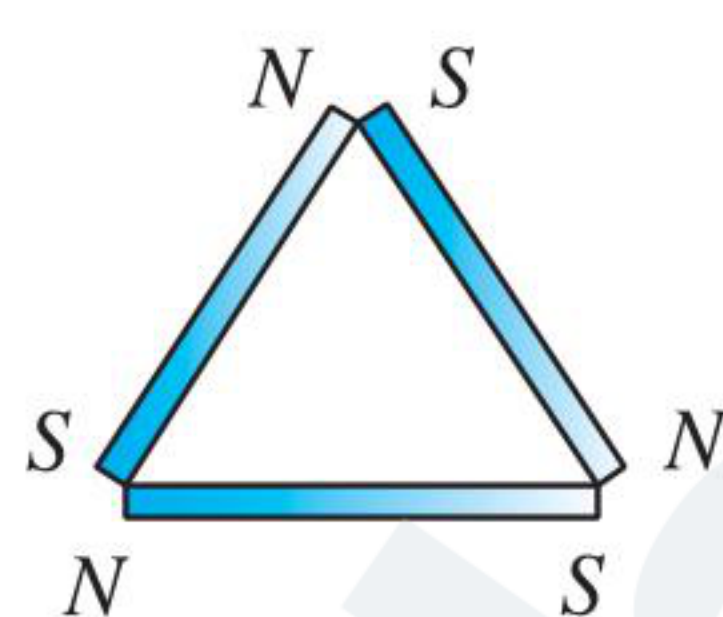
ANSWERS

- 20 Am^{-1}
- (a) $5 \times 10^{-4} \text{ T mA}^{-1}$; (b) 397.9; (c) 396.9; (d) $7.94 \times 10^5 \text{ Am}^{-1}$
- 596.8, $9.534 \times 10^5 \text{ Am}^{-1}$, 595.8
- 684.5
- (a) 596.8; (b) 595.8; (c) $2,000 \text{ Am}^{-1}$; (d) $1.192 \times 10^6 \text{ Am}^{-1}$
- 250 K
- $8.64 \times 10^4 \text{ J}$
- (a) $1.58 \times 10^6 \text{ Am}^{-1}$ (b) 1.985 T

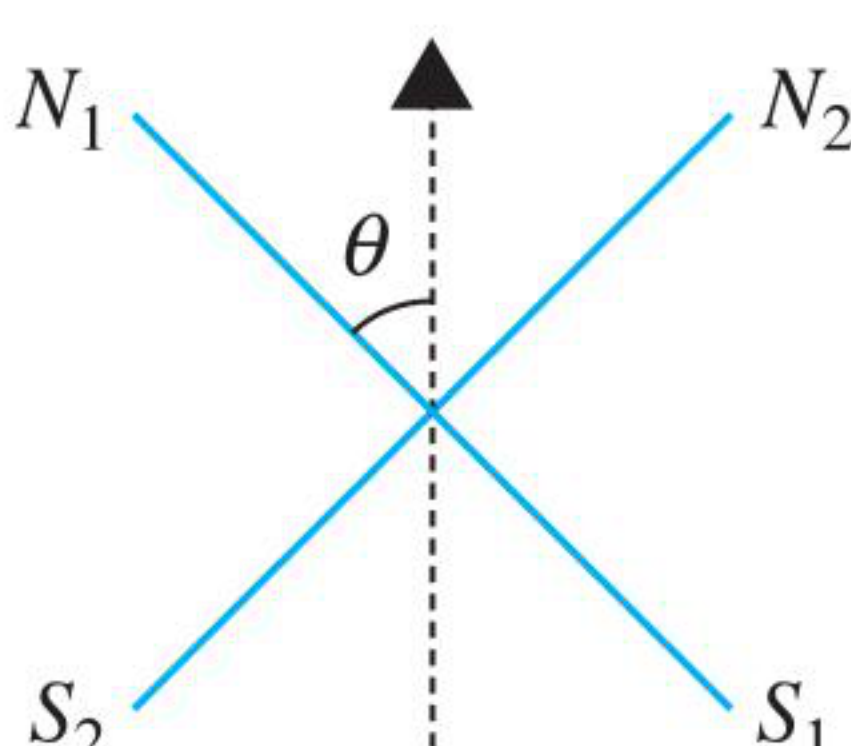
Exercises

Single Correct Answer Type

- A magnet of pole strength m is divided into four equal parts so that the length and breadth of each part is half that of the original magnet. Then pole strength of each of the magnet is
 (1) m (2) $m/4$
 (3) $m/2$ (4) $4m$
- A magnetised wire of moment M is bent into an arc of a circle subtending an angle 60° at the centre, then the new magnetic moment is
 (1) $(2M/\pi)$ (2) (M/π)
 (3) $(3\sqrt{2} M/\pi)$ (4) $(3M/\pi)$
- A thin bar magnet of length $2L$ is bent at the mid-point so that the angle between them is 60° . The new length of the magnet is
 (1) $\sqrt{2} L$ (2) $\sqrt{3} L$
 (3) $2L$ (4) L
- Three identical bar magnets, each of magnetic moment M , are placed in the form of an equilateral triangle with north pole of one touching the south pole of the other (see figure). The net magnetic moment of the system is
 (1) zero (2) $3M$
 (3) $3M/2$ (4) $M\sqrt{3}$
- The B - H curves (a) and (b) shown in the figure are associated with



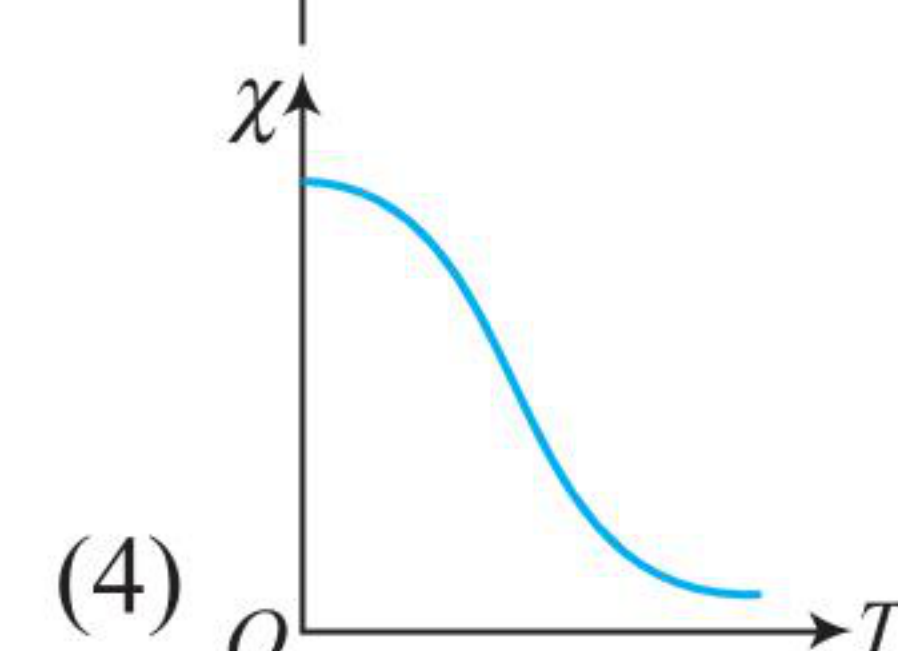
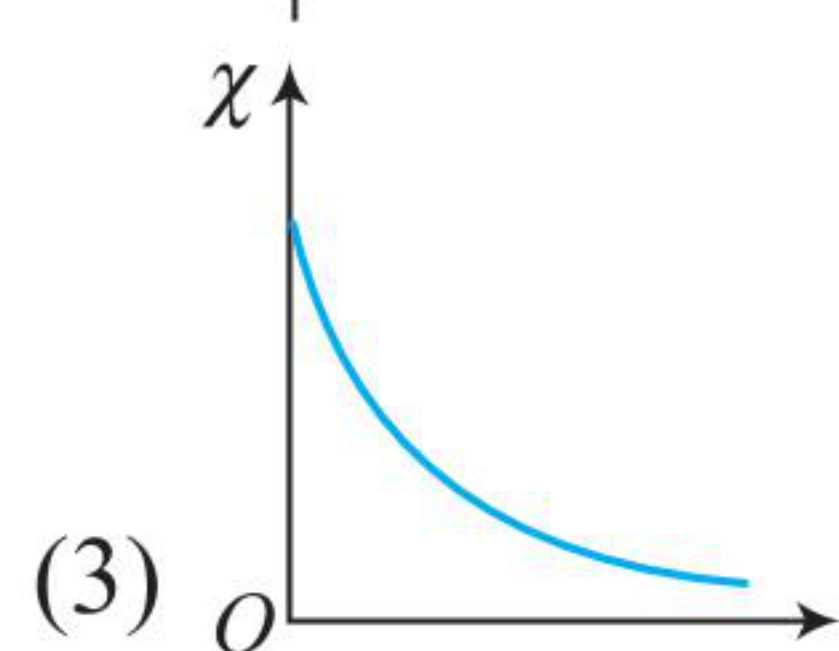
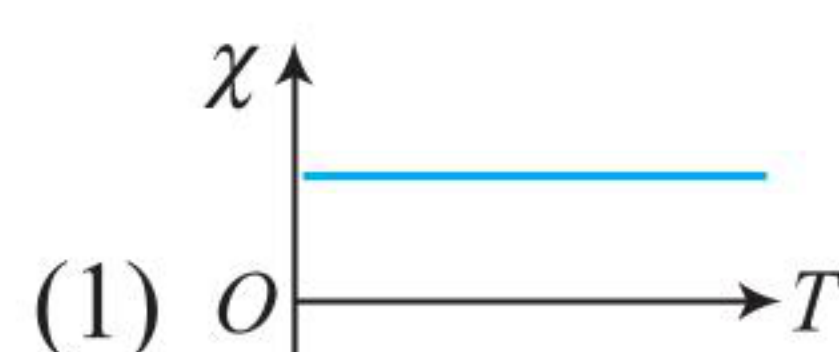
- (1) a diamagnetic and a paramagnetic substance, respectively
 (2) a paramagnetic and a ferromagnetic substance, respectively
 (3) soft iron and steel, respectively
 (4) steel and soft iron, respectively
- Two magnets of equal mass are joined at right angles to each other. Magnet N_1S_1 has a magnetic moment $\sqrt{3}$ times that of N_2S_2 . This arrangement is pivoted so that it is free to rotate in a horizontal plane. When in equilibrium, θ is
 (1) 0° (2) 30°
 (3) 45° (4) 60°
- When a bar magnet is placed at 90° to a uniform magnetic field, it is acted upon by a couple which is maximum. For



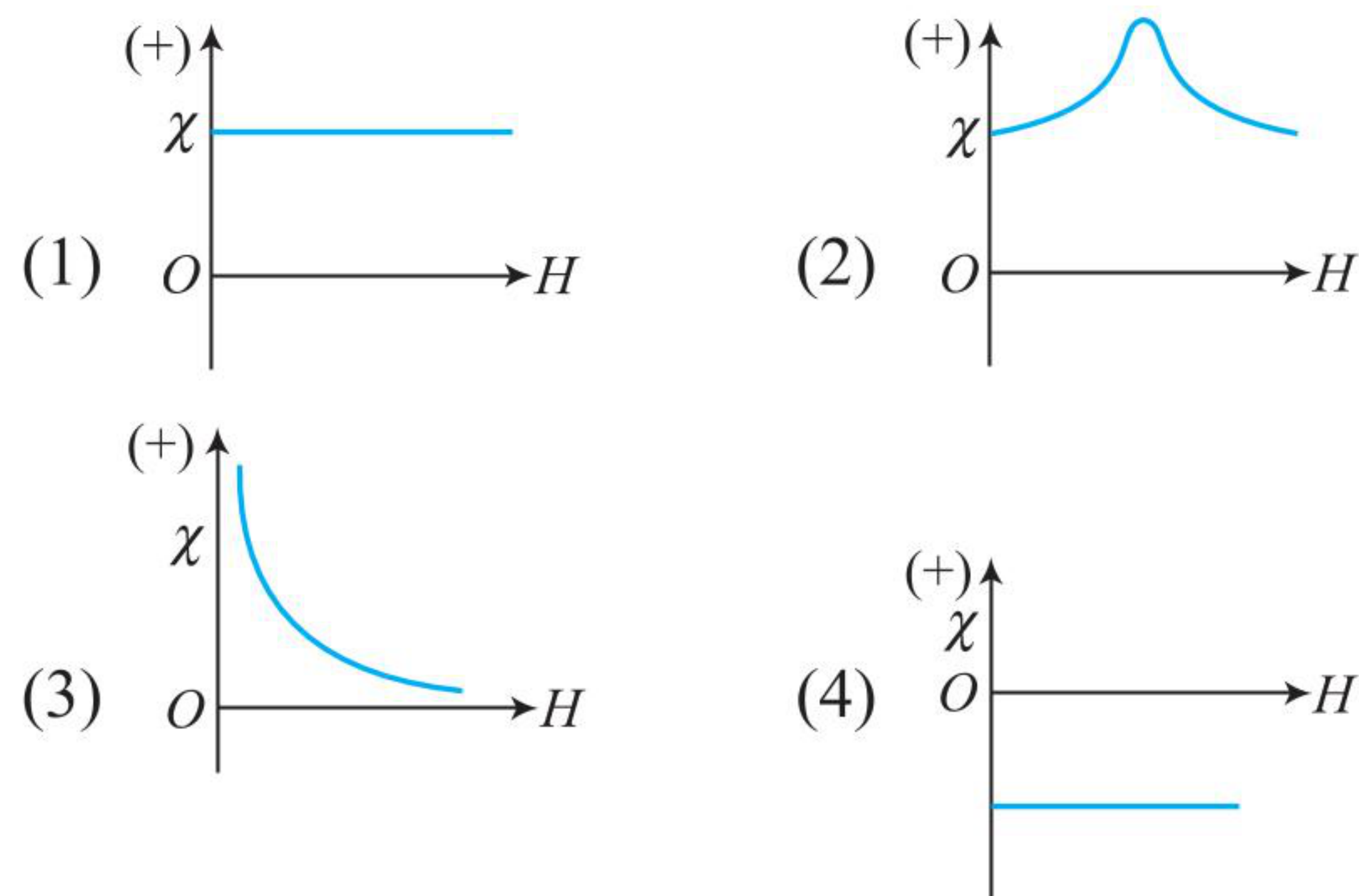
the couple to half the maximum value, the magnet should be inclined to the magnetic field at an angle of

- (1) 45° (2) 30°
 (3) 15° (4) 0°
- The pole strength of a bar magnet is 48 Am and the distance between its poles is 25 cm. The moment of the couple by which it can be placed at an angle of 30° with the uniform magnetic field of flux density $0.15 \text{ N A}^{-1} \text{ m}^{-1}$ will be
 (1) 12 Nm (2) 18 Nm
 (3) 0.9 Nm (4) None of the above
- The distance of two points on the axis of a magnet from its centre are 10 cm. The ratio of magnetic fields at these points is 12.5:1. The length of the magnet is
 (1) 5 cm (2) 25 cm
 (3) 10 cm (4) 20 cm.
- Two identical thin bar magnets each of length l and pole strength m are placed at right angles to each other with north pole of one touching south pole of other, the magnetic moment of the system is
 (1) ml (2) $2ml$
 (3) $\sqrt{2} ml$ (4) $ml/2$
- The work done to turn a magnet by 60° from its equilibrium position is W in a uniform field. The torque required to hold it in that position will be
 (1) $\frac{W}{2}$ (2) $\frac{W}{\sqrt{3}}$
 (3) $\frac{\sqrt{3}}{2} W$ (4) $W\sqrt{3}$
- A compass needle of magnetic moment 60 Am^2 pointing geographical north at a place where the horizontal component of earth field is $40 \mu\text{Wbm}^{-2}$ experiences a torque of $1.2 \times 10^{-3} \text{ Nm}$.
 (1) 15° (2) 30°
 (3) 45° (4) 60°
- In the above question, if the coil is in the magnetic field of 1.2 T, the field being in the plane of coil, then the torque acting on it is
 (1) 1.42 Nm (2) 2.84 Nm
 (3) zero Nm (4) 0.71 Nm
- The force between two short bar magnets with magnetic moments M_1 and M_2 whose centres are r metre apart is 0.8 N when their axes are in the same line. If the separation is increased to $2r$, then force between them is reduced to
 (1) 4.0 N (2) 2.0 N
 (3) 1.0 N (4) 0.5 N
- A magnet is parallel to uniform magnetic field. If it is rotated by 60° , the work done is 0.8 J. How much work is done in moving it 30° further?
 (1) 0.8 ergs (2) 0.4 J
 (3) 8 J (4) 0.8×10^7 ergs

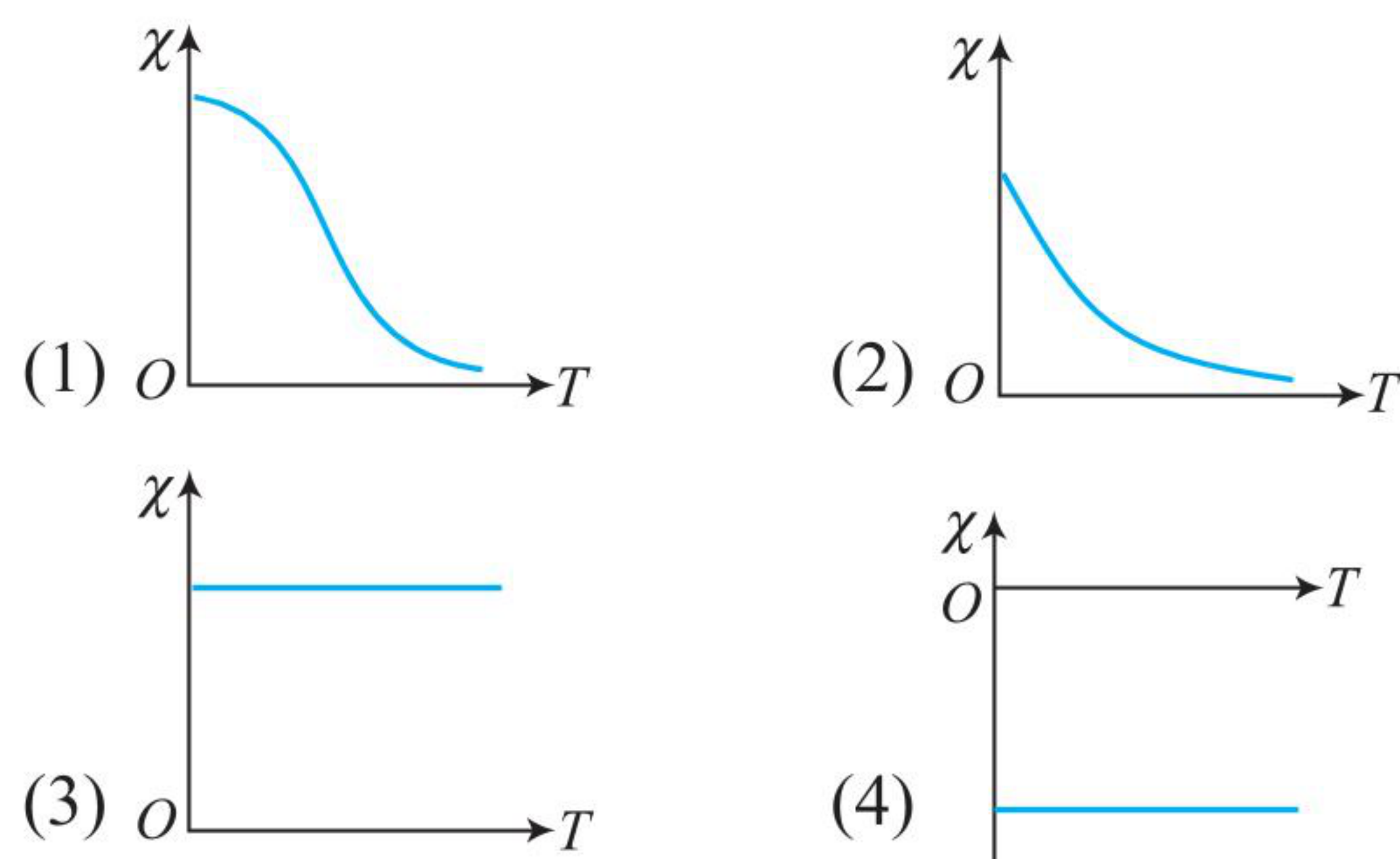
16. In a vibration magnetometer, the time period of a bar magnet oscillating in horizontal component of earth's magnetic field is 2 s. When a magnet is brought near and parallel to it, the time period reduces to 1 s. The ratio H/F of the horizontal component H_a and the field F due to magnet will be
 (1) 3 (2) $1/3$
 (3) $\sqrt{3}$ (4) $1/\sqrt{3}$
17. The magnetic needle of an oscillation magnetometer makes 0 oscillations per minute under the action of earth's magnetic field only. When a bar magnet is placed at some distance along the axis of the needle, it makes 14 oscillations per minute. If the bar magnet be turned so that its poles interchange the positions, the new frequency of the needle is
 (1) 2 oscillations per minute
 (2) 12 oscillations per minute
 (3) 20 oscillations per minute
 (4) 69.7 oscillations per minute
18. A magnetic needle of magnetic moment 60 Am^2 is directed towards the geographical north at a place. It experiences a torque of $1.2 \times 10^{-3} \text{ Nm}$. If the earth's horizontal component of that place is $40 \mu\text{Wbm}^{-2}$ then the angle of declination at that place will be
 (1) 30° (2) 45°
 (3) 60° (4) 90°
19. Two bar magnets of the same mass, same length and breadth but having magnetic moments M and $2M$ are joined together pole for pole and suspended by a string. The time period of assembly in a magnetic field of strength H is 3 seconds. If now the polarity of one of the magnets is reversed and the combination is again made to oscillate in the same field, the time of oscillation is
 (1) $\sqrt{3}$ (2) $3\sqrt{3}$
 (3) 3 (4) 6
20. A magnetic needle vibrates in a vertical plane parallel to the magnetic meridian about the horizontal axis. Its frequency is n . If the plane of oscillation is turned about a vertical axis by 90° , the frequency of oscillation will be
 (1) n (2) zero
 (3) $<n$ (4) $>n$
21. A magnet needle makes 4 vibrations per second at a place where $H = 3.5 \times 10^{-5} \text{ T}$. What is the value of H at place, where the same needle makes 3 vibrations per second?
 (1) $1.98 \times 10^{-5} \text{ T}$ (2) $3.5 \times 10^{-5} \text{ T}$
 (3) $4.0 \times 10^{-5} \text{ T}$ (4) $3.96 \times 10^{-5} \text{ T}$
22. Two bar magnets when placed with their similar poles together makes 20 vib/min. When one of them is reversed the number of vibrations become 15 vib/min. What is the ratio of their dipole moment?
 (1) 7 : 45 (2) 25 : 7
 (3) 1 : 4 (4) 4 : 1
23. A magnet oscillates in earth's field with a time period T . If its mass is quadrupled, then its new time period will be
 (1) $4T$
 (2) $2T$
 (3) $T/2$
 (4) unaffected but motion is not SHM
24. A magnet is cut into four equal parts by cutting it parallel to its length. What will be the time period of each part, if the time period of the original magnet in the same field is T_0 ?
 (1) $\frac{T_0}{\sqrt{2}}$ (2) $\frac{T_0}{2}$
 (3) $\frac{T_0}{4}$ (4) $4T_0$
25. A magnet is suspended in such a way that it oscillates in the horizontal plane. It makes 20 oscillations per minute at a place where dip angle is 30° and 15 oscillations per minute at a place where dip angle is 60° . The ratio of earth's magnetic field at two places is
 (1) $3\sqrt{3} : 8$ (2) $16 : 9\sqrt{3}$
 (3) $4 : 9$ (4) $2\sqrt{2} : 3$
26. Which of the following relation is correct in magnetism?
 (1) $I^2 = V^2 + H^2$ (2) $I = V + H$
 (3) $V = I^2 + H^2$ (4) $V^2 = I + H^2$
27. At a certain place, the horizontal component B_0 and the vertical component V_0 of the earth's magnetic field are equal in magnitude. The total intensity at the place will be
 (1) B_0 (2) B_0^2
 (3) $2B_0$ (4) $\sqrt{2}B_0$
28. A dip circle lies initially in the magnetic meridian. If it is now rotated through angle θ in the horizontal plane, then tangent of the angle of dip is changed in the ratio:
 (1) $1 : \cos \theta$ (2) $\cos \theta : 1$
 (3) $1 : \sin \theta$ (4) $\sin \theta : 1$
29. A current carrying coil is placed with its axis perpendicular to N-S direction. Let horizontal component of earth's magnetic field be H_0 and magnetic field inside the loop is H . If a magnet is suspended inside the loop, it makes angle θ with H . Then $\theta =$
 (1) $\tan^{-1}\left(\frac{H_0}{H}\right)$ (2) $\tan^{-1}\left(\frac{H}{H_0}\right)$
 (3) $\text{cosec}^{-1}\left(\frac{H}{H_0}\right)$ (4) $\cot^{-1}\left(\frac{H_0}{H}\right)$
30. If a magnet is suspended at an angle 30° to the magnetic meridian, it makes an angle of 45° with the horizontal. The real dip is
 (1) $\tan^{-1}(\sqrt{3}/2)$ (2) $\tan^{-1}(\sqrt{3})$
 (3) $\tan^{-1}(\sqrt{3}/2)$ (4) $\tan^{-1}(2/\sqrt{3})$
31. The variation of magnetic susceptibility (χ) with temperature for a diamagnetic substance is best represented by



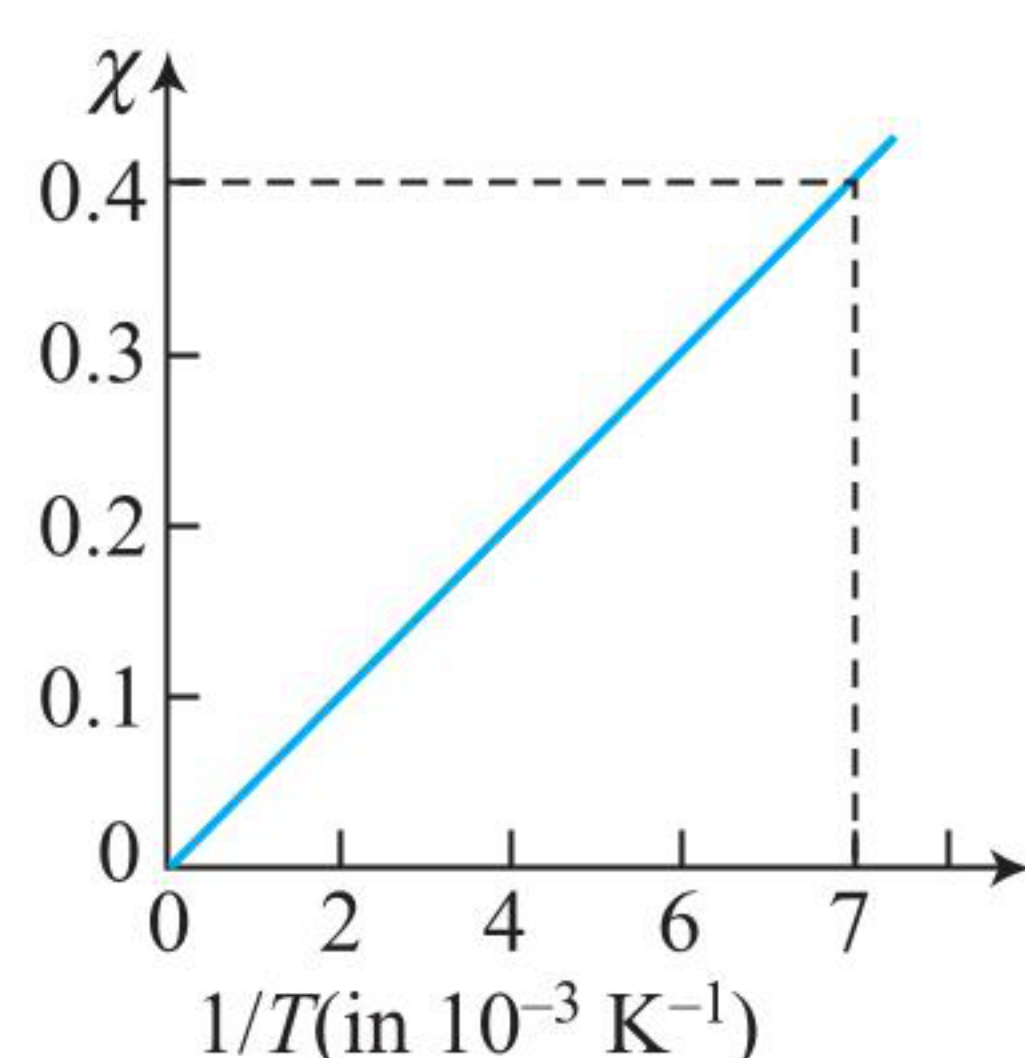
32. The variation of magnetic susceptibility (χ) with magnetising field for a paramagnetic substance is



33. The variation of magnetic susceptibility (χ) with absolute temperature T for a ferromagnetic material is

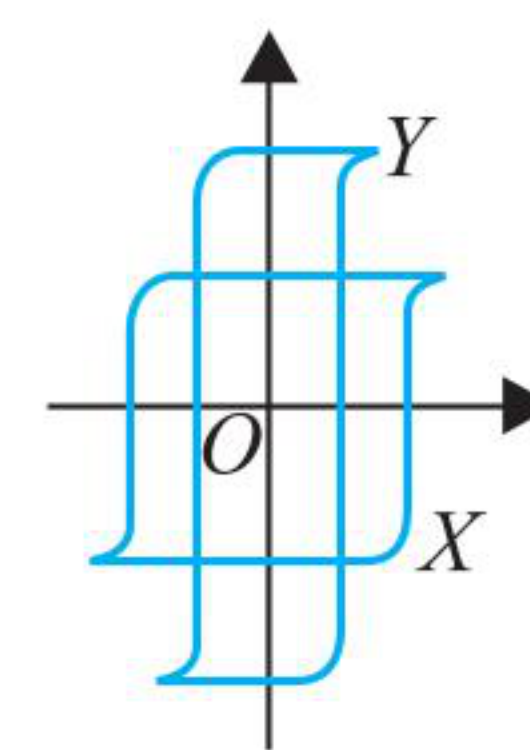


34. The $\chi-1/T$ graph for an alloy of paramagnetic nature is shown in the figure. The Curie constant is, then



- (1) 57 K (2) 2.8×10^{-3} K
(3) 570 K (4) 17.5×10^{-3} K
35. Which of the following is true?
(1) Diamagnetism is temperature dependent
(2) Paramagnetic is temperature dependent
(3) Paramagnetic is temperature independent
(4) None of these
36. If a magnetic substance is kept in a magnetic field, then which of the following is thrown out?
(1) Paramagnetic (2) Ferromagnetic
(3) Diamagnetic (4) Antiferromagnetic
37. A paramagnetic material is kept in a magnetic field. The field is increased till the magnetisation becomes constant. If the temperature is now decreased, the magnetisation
(1) will increase (2) decreases
(3) remain constant (4) may increase or decrease
38. The susceptibility of a ferromagnetic material is K at 27°C . At what temperature will its susceptibility be $0.5 K$?
(1) 54°C (2) 327°C
(3) 600°C (4) 237°C

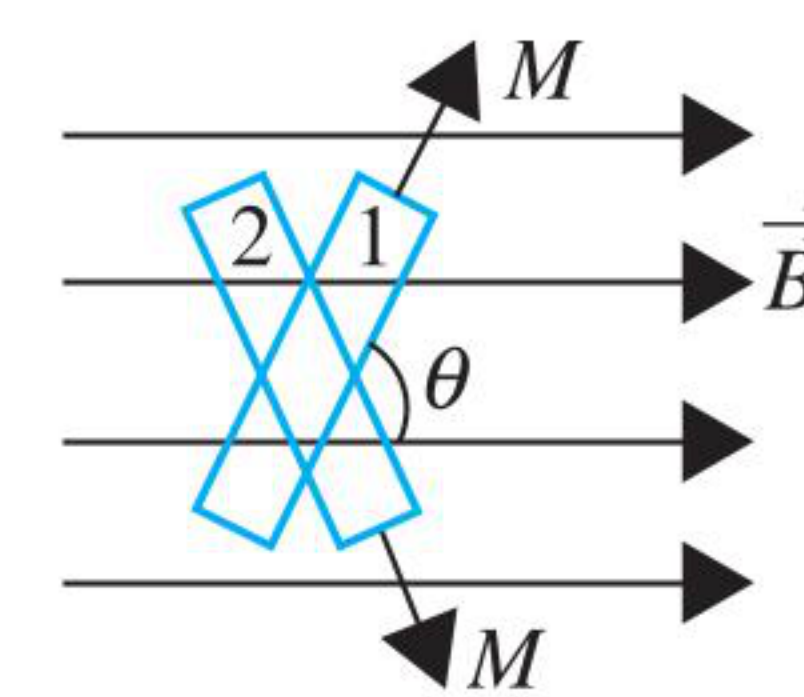
39. Two ferromagnetic materials X and Y have hysteresis curves of the shapes shown (see figure)



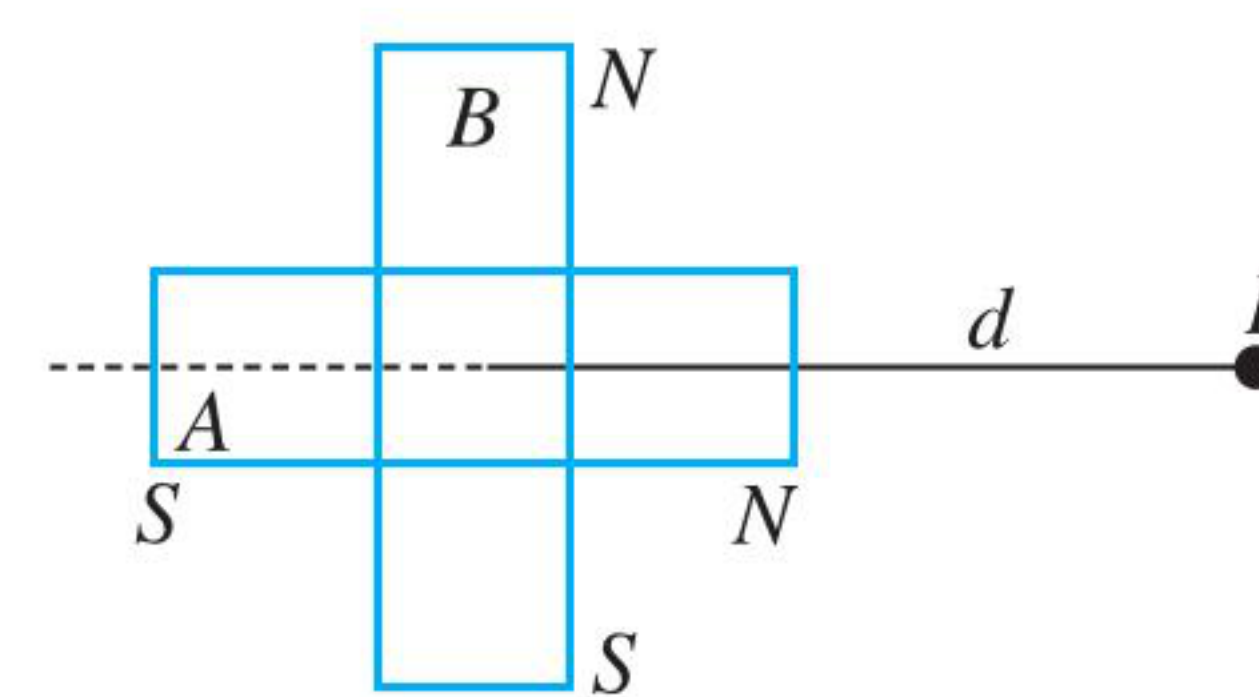
- (1) X and Y are suitable for making a permanent magnet as well as an electromagnet.
(2) X is more suitable for making a permanent magnet while Y is more suitable for making an electromagnet.
(3) X is more suitable for making an electromagnet while Y is more suitable for making a permanent magnet.
(4) neither X nor Y is suitable for making either a permanent magnet or an electromagnet.
40. A bar magnet is suspended horizontally by a torsionless wire in the magnetic meridian. In order to deflect the magnet through 30° from the magnetic meridian, the upper end of the wire has to be rotated by 180° . Now this magnet is replaced by another magnet. If it has to be deflected by 30° from the magnetic meridian, upper end of the wire has to be rotated by 270° . Ratio of the magnetic moments of two magnets is

- (1) $\frac{8}{5}$ (2) $\frac{7}{6}$
(3) $\frac{6}{7}$ (4) $\frac{5}{8}$

41. M and $M\sqrt{3}$ are the magnetic dipole moments of the two magnets which are joined to form a cross figure. The inclination of the system with the field, if their combination is suspended freely in a uniform external magnetic field B is
(1) $\theta = 30^\circ$ (2) $\theta = 45^\circ$
(3) $\theta = 60^\circ$ (4) $\theta = 15^\circ$

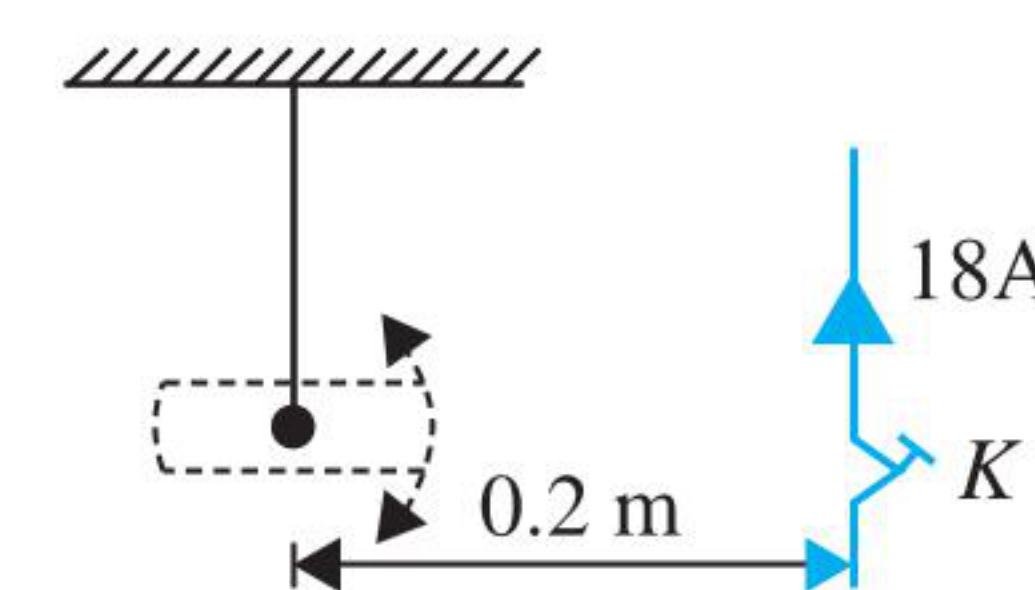


42. The magnetic induction at P , for the arrangement shown in the figure, when two similar short magnets of magnetic moment M are joined at the middle so that they are mutually perpendicular will be



- (1) $\frac{\mu_0 M\sqrt{3}}{4\pi d^3}$ (2) $\frac{\mu_0 3M}{4\pi d^3}$
(3) $\frac{\mu_0 M\sqrt{5}}{4\pi d^3}$ (4) $\frac{\mu_0 2M}{4\pi d^3}$

43. Figure shows a short magnet executing small oscillations in a vibration magnetometer in earth's magnetic field having horizontal component $24 \mu\text{T}$. The period of oscillation is 0.1 s. When the key K is closed, an upward current of 18 A is established as shown. The new time period is
(1) 0.1 s (2) 0.2 s
(3) 0.3 s (4) 0.4 s



44. A bar of mass M is suspended by two wires. Assume that a uniform magnetic field B is directed into the page. When the current through the bar is I , then the tension in each supporting wire is
- (1) $\frac{Mg}{2}$ (2) $2BIL$
 (3) $Mg - BIL$ (4) $\frac{Mg - BIL}{2}$
45. A coil in the shape of an equilateral triangle of side 0.02 m is suspended from its vertex such that it is hanging in a vertical plane between the pole pieces of permanent magnet producing a uniform field of 5×10^{-2} T. If a current of 0.1 A is passed through the coil, what is the couple acting?
- (1) $5\sqrt{3} \times 10^{-7}$ N-m (2) $5\sqrt{3} \times 10^{-10}$ N-m
 (3) $\frac{\sqrt{3}}{5} \times 10^{-7}$ N-m (4) None of these
46. A vibration magnetometer consists of two identical bar magnets placed one over the other such that they are mutually perpendicular and bisect each other. The time period of combination is 4 s. If one of the magnets is removed, find the period of other
- (1) 5 s (2) 3.36 s
 (3) 4.36 s (4) 5.36 s
47. A short bar magnet of magnetic moment 255 JT^{-1} is placed with its axis perpendicular to earth's field direction. At what distance from the centre of the magnet, the resultant field is inclined at 45° with earth's field, $H = 0.4 \times 10^{-4}$ T.
- (1) 5 m (2) 0.5 m
 (3) 2.5 m (4) 0.25 m
48. A magnetic dipole is under the effect of two magnetic fields inclined at 75° to each other. One of the fields has a magnitude of 1.5×10^{-2} T. The magnets come to stable position at an angle of 30° with the direction of the above field. The magnitude of other field is
- (1) $\frac{1.5}{2\sqrt{2}} \times 10^{-2}$ T (2) $\frac{1.5}{\sqrt{2}} \times 10^{-2}$ T
 (3) $1.5 \times \sqrt{2} \times 10^{-2}$ T (4) 1.5×10^{-2} T
49. The dip at a place is δ . For measuring it, the axis of the dip needle is perpendicular to the magnetic meridian. If the axis of the dip needle makes angle θ with the magnetic meridian, the apparent dip will be given $\tan \delta_1$ which is equal to
- (1) $\tan \delta \cos \theta$ (2) $\tan \delta \sec \theta$
 (3) $\tan \delta \sin \theta$ (4) $\tan \delta \operatorname{cosec} \theta$
50. The dipole moment of each molecule of a paramagnetic gas is $1.5 \times 10^{-23} \text{ A} \times \text{m}^2$. The temperature of gas is 27°C and the number of molecules per unit volume in it is $2 \times 10^{26} \text{ m}^{-3}$. The maximum possible intensity of magnetisation in the gas will be
- (1) $3 \times 10^3 \text{ A/m}$ (2) $4 \times 10^{-3} \text{ A/m}$
 (3) $5 \times 10^5 \text{ A/m}$ (4) $6 \times 10^{-4} \text{ A/m}$
51. Each atom of an iron bar ($5 \text{ cm} \times 1 \text{ cm} \times 1 \text{ cm}$) has a magnetic moment $1.8 \times 10^{-23} \text{ A m}^2$, knowing that the density of iron is $7.78 \times 10^3 \text{ kg m}^{-3}$, atomic weight is 56 and Avogadro's number is 6.02×10^{23} , the magnetic moment of bar in the state of magnetic saturation will be
- (1) 4.75 Am^2 (2) 5.74 Am^2
 (3) 7.54 Am^2 (4) 75.4 Am^2
52. An iron rod of volume 10^{-4} m^3 and relative permeability 1000 is placed inside a long solenoid wound with 5 turns/cm. If a current of 0.5 A is passed through the solenoid, then the magnetic moment of the rod is
- (1) 10 Am^2 (2) 15 Am^2
 (3) 20 Am^2 (4) 25 Am^2
53. A bar magnet has coercivity $4 \times 10^3 \text{ Am}^{-1}$. It is desired to demagnetise it by inserting it inside a solenoid 12 cm long and having 60 turns. The current that should be sent through the solenoid is
- (1) 2 A (2) 4 A
 (3) 6 A (4) 8 A
54. The magnetic moment produced in a substance of 1 g is 6×10^{-7} ampere-metre². If its density is 5 g/cm^3 , then the intensity of magnetisation in A/m will be
- (1) 8.3×10^6 (2) 3.0
 (3) 1.2×10^{-7} (4) 3×10^{-6}
55. The area of hysteresis loop of a material is equivalent to 250 joule. When 10 kg material is magnetised by an alternating field of 50 Hz then energy lost in one hour will be if the density of material is 7.5 g/cm^3
- (1) $6 \times 10^4 \text{ J}$ (2) $6 \times 10^4 \text{ erg}$
 (3) $3 \times 10^2 \text{ J}$ (4) $3 \times 10^2 \text{ erg}$
56. The magnet of vibration magnetometer is heated so as to reduce its magnetic moment by 36%. By doing this the periodic time of the magnetometer will
- (1) increases by 36% (2) increases by 25%
 (3) decreases by 25% (4) decreases by 64%
57. A magnetic needle lying parallel to a magnetic field requires W units of work to turn it through 60° . The torque required to maintain the needle in this position will be
- (1) $\sqrt{3} W$ (2) W
 (3) $\frac{\sqrt{3}}{2} W$ (4) $2W$
58. Two similar bar magnets P and Q , each of magnetic moment M , are taken. If P is cut along its axial line and Q is cut along its equatorial line, all the four pieces obtained have
- (1) equal pole strength (2) magnetic moment $M/4$
 (3) magnetic moment $M/2$ (4) magnetic moment M
59. A bar magnet is held perpendicular to a uniform magnetic field. If the couple acting on the magnet is to be halved by rotating it, then the angle by which it is to be rotated is
- (1) 30° (2) 45°
 (3) 60° (4) 90°
60. A current carrying coil is placed with its axis perpendicular to N-S direction. Let horizontal component of earth's magnetic field be H_0 and magnetic field inside the loop is H . If a magnet is suspended inside the loop, it makes angle θ with H . Then $\theta =$
- (1) $\tan^{-1} \left(\frac{H_0}{H} \right)$ (2) $\tan^{-1} \left(\frac{H}{H_0} \right)$
 (3) $\operatorname{cosec}^{-1} \left(\frac{H}{H_0} \right)$ (4) $\cot^{-1} \left(\frac{H_0}{H} \right)$

61. A small rod of bismuth is suspended freely between the poles of a strong electromagnet. It is found to arrange itself at right angles to the magnetic field. This observation establishes that bismuth is

- (1) diamagnetic (2) paramagnetic
(3) ferri-magnetic (4) antiferro-magnetic

62. Two identical magnetic dipoles of magnetic moments 1.0 A-m^2 each, placed at a separation of 2 m with their axis perpendicular to each other. The resultant magnetic field at a point midway between the dipoles is

- (1) $5 \times 10^{-7} \text{ T}$ (2) $\sqrt{5} \times 10^{-7} \text{ T}$
(3) 10^{-7} T (4) None of these

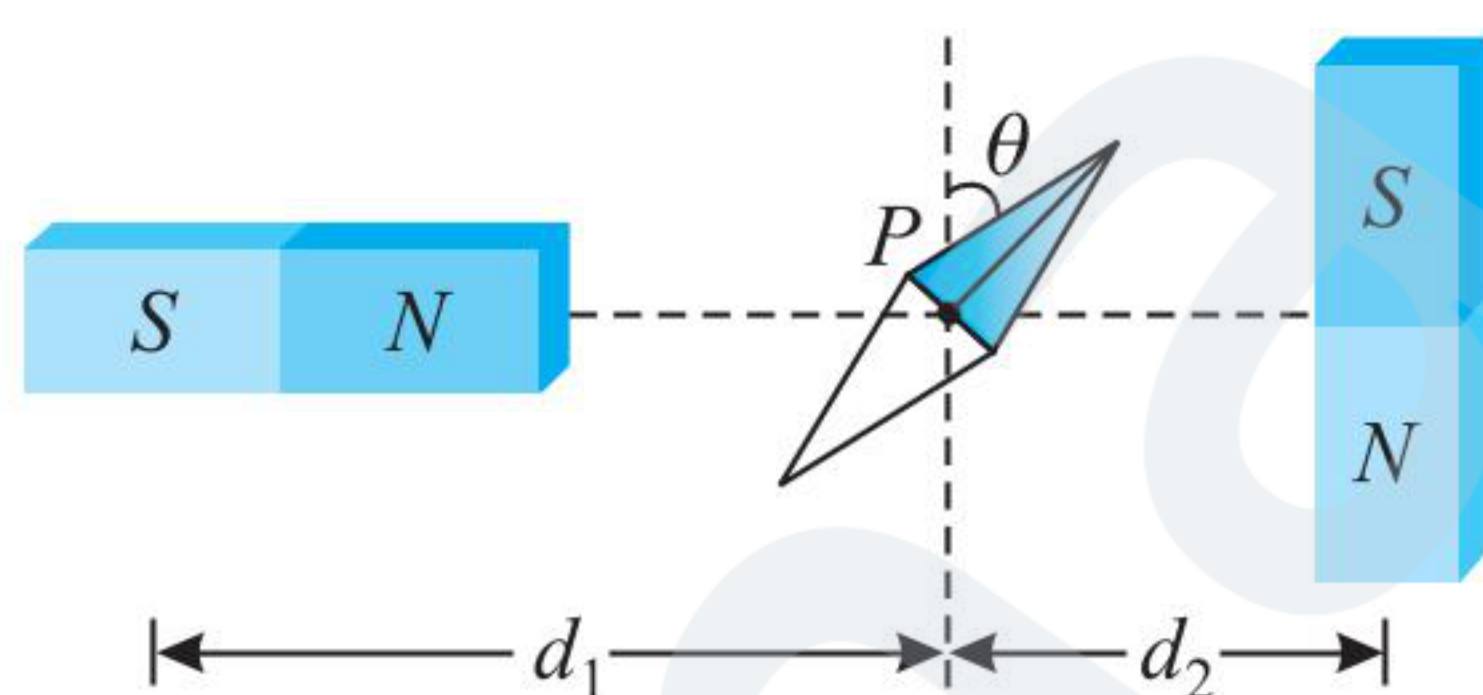
63. Two identical short bar magnets, each having magnetic moment M , are placed a distance of $2d$ apart with axes perpendicular to each other in a horizontal plane. The magnetic induction at a point midway between them is

- (1) $\frac{\mu_0}{4\pi}(\sqrt{2})\frac{M}{d^3}$ (2) $\frac{\mu_0}{4\pi}(\sqrt{3})\frac{M}{d^3}$
(3) $\left(\frac{2\mu_0}{\pi}\right)\frac{M}{d^3}$ (4) $\frac{\mu_0}{4\pi}(\sqrt{5})\frac{M}{d^3}$

64. A short bar magnet with its north pole facing north forms a neutral point at P in the horizontal plane. If the magnet is rotated by 90° in the horizontal plane, the net magnetic induction at P is (Horizontal component of earth's magnetic field = B_H)

- (1) 0 (2) $2 B_H$
(3) $\frac{\sqrt{5}}{2} B_H$ (4) $\sqrt{5} B_H$

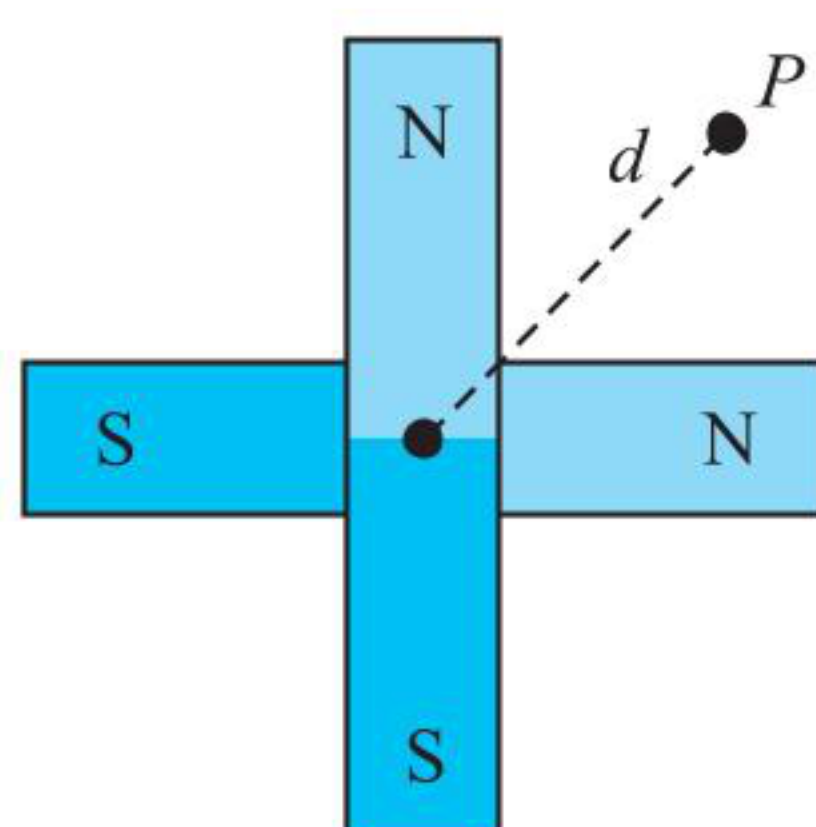
65. Two magnets A and B are identical and these are arranged as shown in the figure. Their length is negligible in comparison to the separation between them. A magnetic needle is placed between the magnets at point P which gets deflected through an angle θ under the influence of magnets. The ratio of distance d_1 and d_2 will be



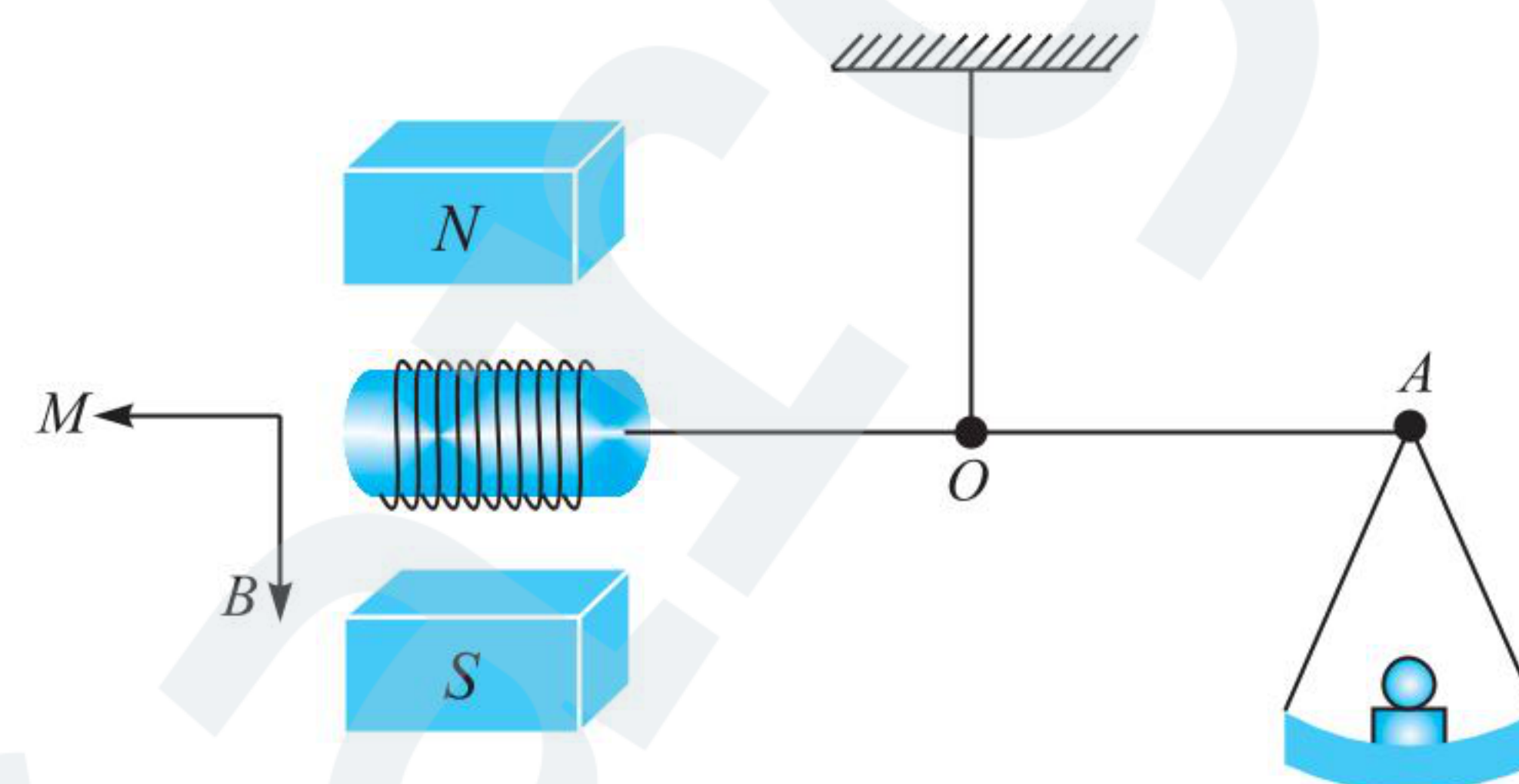
- (1) $(2 \tan \theta)^{1/3}$ (2) $(2 \tan \theta)^{-1/3}$
(3) $(2 \cot \theta)^{1/3}$ (4) $(2 \cot \theta)^{-1/3}$

66. Two short magnets of equal dipole moments M are fastened perpendicularly at their centre (see figure). The magnitude of the magnetic field at a distance d from the centre on the bisector of the right angle is

- (1) $\frac{\mu_0}{4\pi} \frac{M}{d^3}$ (2) $\frac{\mu_0}{4\pi} \frac{M\sqrt{2}}{d^3}$
(3) $\frac{\mu_0}{4\pi} \frac{2\sqrt{2}M}{d^3}$ (4) $\frac{\mu_0}{4\pi} \frac{2M}{d^3}$



67. A small coil C with $N = 200$ turns is mounted on one end of a balance beam and introduced between the poles of an electromagnet as shown in the figure. The cross-sectional area of coil is $A = 1.0 \text{ cm}^2$, length of arm OA of the balance beam is $l = 30 \text{ cm}$. When there is no current in the coil the balance is in equilibrium. On passing a current $I = 22 \text{ mA}$ through the coil the equilibrium is restored by putting the additional counter weight of mass $\Delta m = 60 \text{ mg}$ on the balance pan. Find the magnetic induction at the spot where coil is located.



- (1) 0.4 T (2) 0.3 T
(3) 0.2 T (4) 0.1 T

68. If ϕ_1 and ϕ_2 be the angles of dip observed in two vertical planes at right angles to each other and ϕ be the true angle of dip, then

- (1) $\cos^2 \phi = \cos^2 \phi_1 + \cos^2 \phi_2$
(2) $\sec^2 \phi = \sec^2 \phi_1 + \sec^2 \phi_2$
(3) $\tan^2 \phi = \tan^2 \phi_1 + \tan^2 \phi_2$
(4) $\cot^2 \phi = \cot^2 \phi_1 + \cot^2 \phi_2$

69. A dip needle lies initially in the magnetic meridian when it shows an angle of dip θ at a place. The dip circle is rotated through an angle x in the horizontal plane and then it shows an angle of dip θ' . Then $\frac{\tan \theta'}{\tan \theta}$ is

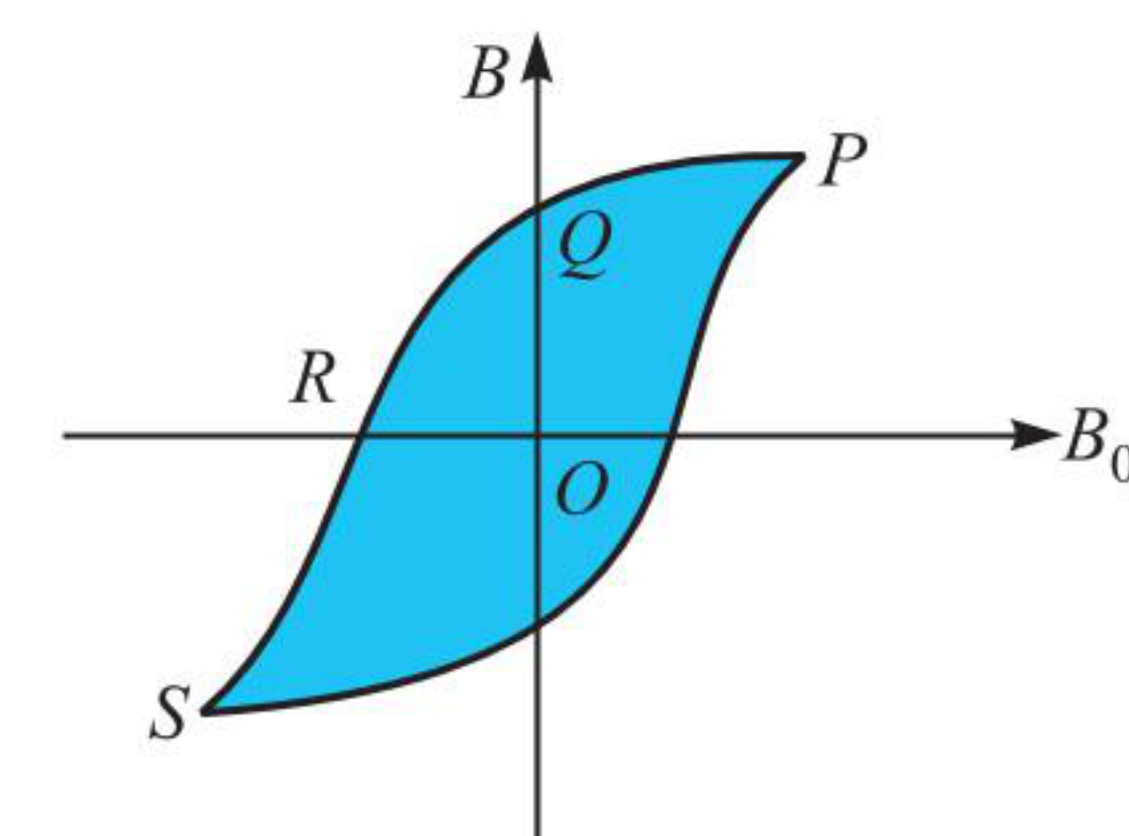
- (1) $\frac{1}{\cos x}$ (2) $\frac{1}{\sin x}$
(3) $\frac{1}{\tan x}$ (4) $\cos x$

70. A dip circle is adjusted so that its needle moves freely in the magnetic meridian. In this position, the angle of dip is 40° . Now the dip circle is rotated so that the plane in which the needle moves makes an angle of 30° with the magnetic meridian. In this position the needle will dip by an angle

- (1) 40° (2) 30°
(3) More than 40° (4) Less than 40°

71. The figure illustrates how B , the flux density, inside a sample of unmagnetised ferromagnetic material varies with B_0 , the magnetic flux density in which the sample is kept. For the sample to be suitable for making a permanent magnet

- (1) OQ should be large, OR should be small
(2) OQ and OR should both be large
(3) OQ should be small and OR should be large
(4) OQ and OR should both be small



Answers Key

EXERCISES

Single Correct Answer Type

1. (3)	2. (4)	3. (4)	4. (1)	5. (3)	31. (2)	32. (1)	33. (1)	34. (1)	35. (2)
6. (2)	7. (2)	8. (3)	9. (3)	10. (3)	36. (3)	37. (3)	38. (2)	39. (2)	40. (4)
11. (4)	12. (2)	13. (1)	14. (4)	15. (4)	41. (3)	42. (3)	43. (2)	44. (4)	45. (1)
16. (2)	17. (1)	18. (1)	19. (2)	20. (3)	46. (2)	47. (2)	48. (2)	49. (2)	50. (1)
21. (1)	22. (2)	23. (2)	24. (1)	25. (2)	51. (3)	52. (4)	53. (4)	54. (2)	55. (1)
26. (1)	27. (4)	28. (1)	29. (1)	30. (1)	56. (2)	57. (1)	58. (3)	59. (3)	60. (1)
					61. (1)	62. (2)	63. (4)	64. (4)	65. (3)
					66. (3)	67. (1)	68. (4)	69. (1)	70. (3)
					71. (2)				

Chapter 3

Concept Application Exercises

Exercise 3.1

1. Here, $F = 5 \text{ gf} = 5 \times 10^{-3} \text{ kgf} = 5 \times 10^{-3} \times 9.8 \text{ N}$;
 $r = 10 \text{ cm} = 0.1 \text{ m}$

Let m and $4m$ be the strengths of two poles.

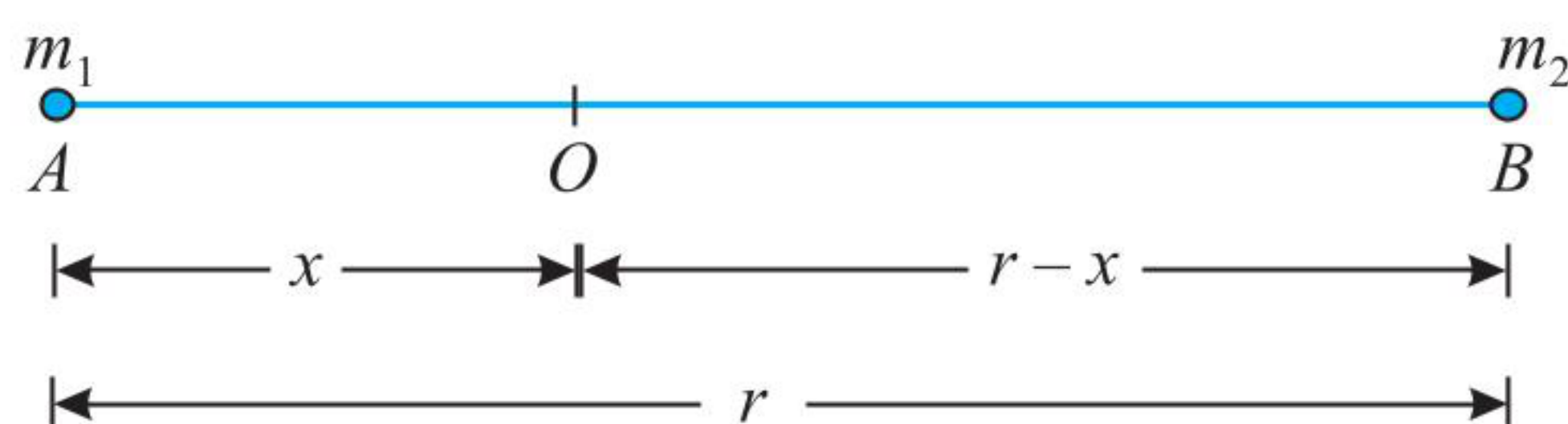
$$\text{Now, } F = \frac{\mu_0}{4\pi} \cdot \frac{m \times 4m}{r^2}$$

$$\text{Or } 5 \times 10^{-3} \times 9.8 = \frac{10^{-7} \times m \times 4m}{(0.1)^2}$$

$$\text{Or } m = 35 \text{ Am}$$

$$\text{and } 4m = 4 \times 35 = 140 \text{ Am}$$

2. Here, $m_1 = 25 \text{ Am}$; $m_2 = 64 \text{ Am}$ and $r = 1.0 \text{ m}$



Let the magnetic field be zero at the point O , such that $AO = x$.

$$\text{Then, } \frac{\mu_0}{4\pi} \cdot \frac{m_1}{x^2} = \frac{\mu_0}{4\pi} \cdot \frac{m_2}{(r-x)^2}$$

$$\text{Or } \frac{25}{x^2} = \frac{64}{(1.0-x)^2}$$

$$\text{Or } 64x^2 = 25(1.0-x)^2$$

$$\text{or } 8x = \pm 5(1.0-x)$$

$$\text{or } 8x = 5(1.0-x) \text{ or } -5(1.0-x)$$

$$\text{or } x = 0.385 \text{ m or } -1.667 \text{ m}$$

3. Since lengths of both the magnets are quite large as compared to separation between them, the interaction is effective only between their N-poles.

Let m be the strength of each pole of the two magnets. Then,

$$\frac{\mu_0}{4\pi} \cdot \frac{m \times m}{r^2} = \text{weight of the magnet}$$

$$\text{Here, weight of the magnet} = 50 \text{ gf} = 50 \times 10^{-3} \text{ kgf}$$

$$= 50 \times 10^{-3} \times 9.8 \text{ N}$$

$$\text{and } r = 3 \text{ mm} = 3 \times 10^{-3} \text{ m}$$

$$\therefore \frac{10^{-7} \times m^2}{(3 \times 10^{-3})^2} = 50 \times 10^{-3} \times 9.8$$

$$\text{or } m = 6.64 \text{ Am}$$

4. Period of revolution of electron,

$$T = \frac{2\pi r}{v} = \frac{2\pi \times 0.53 \times 10^{-10}}{2.3 \times 10^6} = 1.448 \times 10^{-16} \text{ s}$$

The orbital motion of electron is equivalent to current,

$$I = \frac{e}{T} = \frac{1.6 \times 10^{-19}}{1.448 \times 10^{-16}} = 1.105 \times 10^{-3} \text{ A}$$

Therefore, magnetic moment of the revolving electron,

$$M = IA = I \times \pi r^2 = 1.105 \times 10^{-3} \times \pi \times (0.53 \times 10^{-10})^2$$

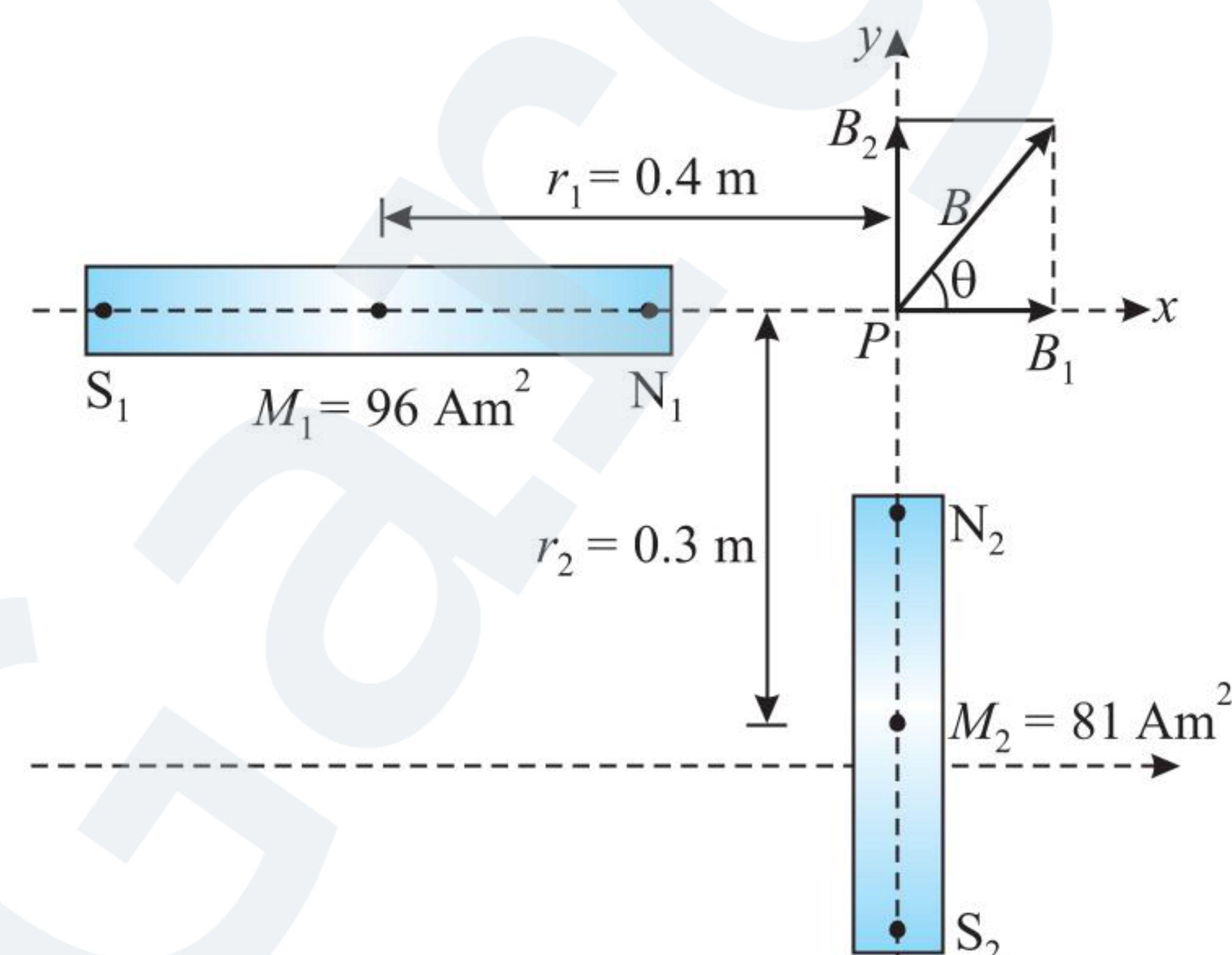
$$= 9.75 \times 10^{-24} \text{ Am}^2$$

5. $M_1 = 4 \times 0.21 = 0.84 \text{ Am}^2$ and $M_2 = 7 \times 0.12 = 0.84 \text{ Am}^2$

Let M be the magnetic moment of the combination of two magnets placed at right angles to each other. Then,

$$M = \sqrt{M_1^2 + M_2^2} = \sqrt{(0.84)^2 + (0.84)^2} = 1.19 \text{ Am}^2$$

6. The two magnets are placed as shown in figure.



The point P lies on axial line of both the magnets.

Let B_1 and B_2 be the magnetic fields at the point P due to the magnets N_1S_1 and N_2S_2 respectively. Then,

$$B_1 = \frac{\mu_0}{4\pi} \cdot \frac{2M_1}{r_1^3} = \frac{10^{-7} \times 2 \times 96}{(0.4)^3}$$

$$= 3 \times 10^{-4} \text{ T} \quad (\text{along } PX)$$

$$\text{and } B_2 = \frac{\mu_0}{4\pi} \cdot \frac{2M_2}{r_2^3} = \frac{10^{-7} \times 2 \times 81}{(0.3)^3}$$

$$= 6 \times 10^{-4} \text{ T} \quad (\text{along } PY)$$

Therefore, resultant magnetic field at the point P ,

$$B = \sqrt{B_1^2 + B_2^2} = \sqrt{(3 \times 10^{-4})^2 + (6 \times 10^{-4})^2}$$

$$= 3\sqrt{5} \times 10^{-4} \text{ T}$$

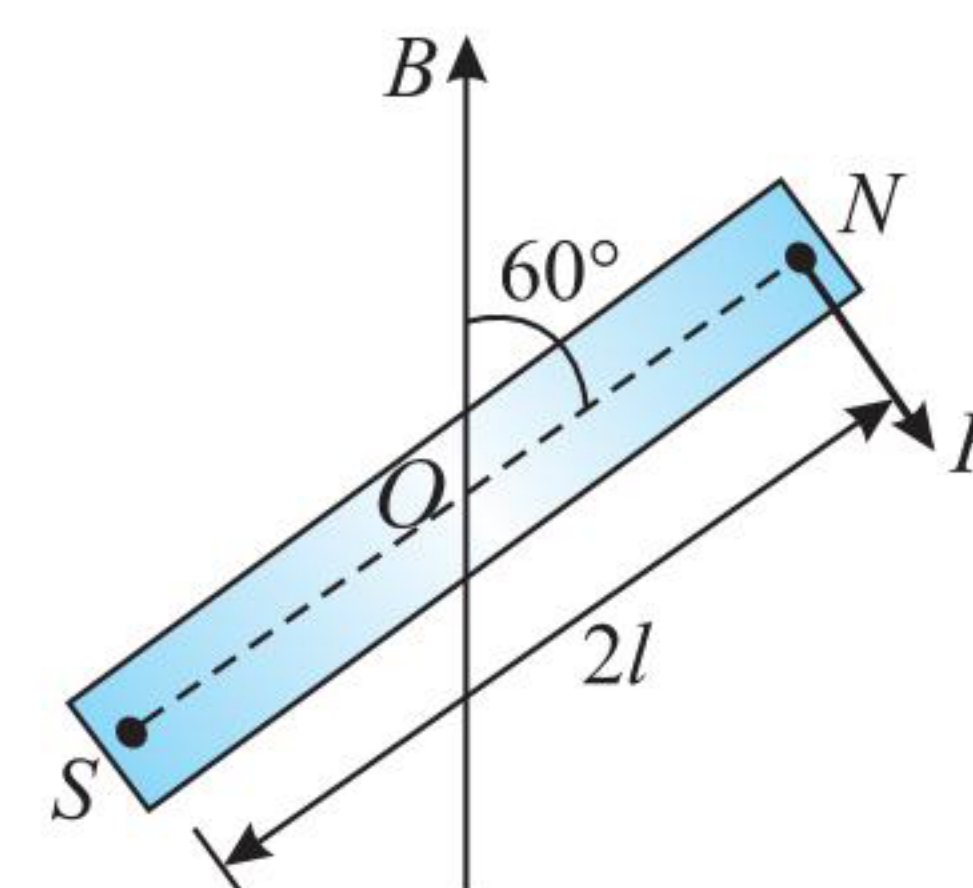
If the direction of B makes an angle θ with the X -axis (or N_1S_1) then

$$\tan \theta = \frac{B_2}{B_1} = \frac{6 \times 10^{-4}}{3 \times 10^{-4}} = 2$$

$$\text{or } \theta = \tan^{-1} 2$$

7. Here, $m = 14.4 \text{ Am}$; $B = 0.25 \text{ T}$; $2l = 25 \text{ cm} = 0.25 \text{ m}$

The magnet is held in equilibrium at 60° to the direction of the magnetic field B by applying force F as shown in figure.



For the equilibrium of the magnet,

$$F \times ON = MB \sin \theta$$

$$\text{Or } F \times l = (m \times 2l) B \sin 60^\circ$$

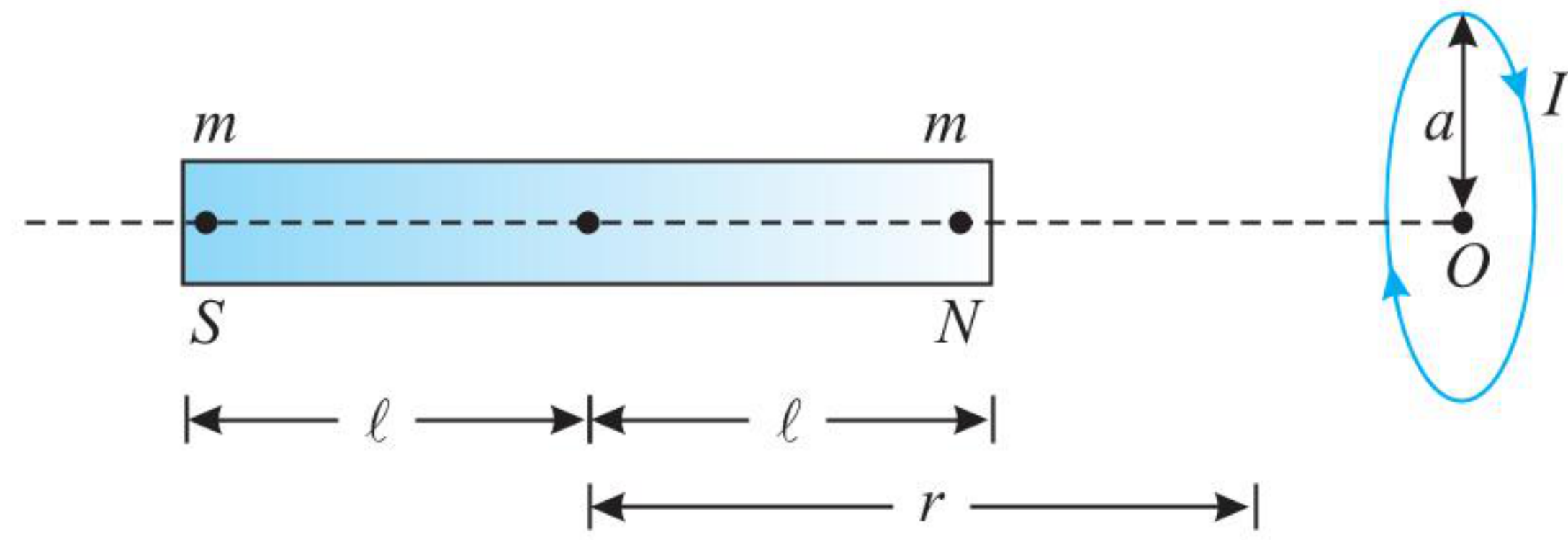
$$\text{Or } F = 2mB \sin 60^\circ$$

$$= 2 \times 14.4 \times 0.25 \times \sin 60^\circ = 6.24 \text{ N}$$

If the force is removed, magnet will tend to align itself along the magnetic field.

8. The circular coil carrying current is placed on the axis of the magnet as shown in figure.

Here, $a = 0.002$ m, $I = 2$ A; $2l = 0.1$ m; $r = 0.15$ m
and $M = 10^5$ JT⁻¹



Therefore, pole strength of the magnet,

$$m = \frac{M}{2l} = \frac{10^5}{0.1} = 10^6 \text{ Am}$$

Magnetic field at a distance x from the centre of a current carrying coil of radius a ,

$$B = \frac{\mu_0}{4\pi} \cdot \frac{2\pi I a^2}{x^3} \quad (a \ll x)$$

If B_1 and B_2 are magnetic fields due to the coil at the north and south poles of the magnet, then force on the magnet due to the coil,

$$F = m B_2 - m B_1$$

According to Newton's third law, magnet will exert an equal and opposite force on the coil. Therefore, force on the coil due to the magnet,

$$\begin{aligned} F &= m B_1 - m B_2 = m(B_1 - B_2) \\ &= m \left[\frac{\mu_0}{4\pi} \cdot \frac{2\pi I a^2}{(2-l)^3} - \frac{\mu_0}{4\pi} \cdot \frac{2\pi I a^2}{(r+l)^3} \right] \\ &= \frac{\mu_0}{4\pi} \cdot 2\pi I a^2 m \left[\frac{1}{(r-l)^3} - \frac{1}{(r+l)^3} \right] \end{aligned}$$

On substituting the corresponding values in above expression, we get, $F = 4.4 \times 10^{-3}$ N

Exercise 3.2

1. Here, $B_H = \sqrt{3} B_V$

$$\text{Now, } \delta = \frac{B_V}{B_H} = \frac{B_V}{\sqrt{3} B_V} = \frac{1}{\sqrt{3}}$$

$$\text{Or } \delta = 30^\circ$$

2. Here, $\delta = 60^\circ$; $B_V = 0.3 \times 10^{-4}$ T

$$\text{Now, } B_H = B \cos \delta$$

$$\begin{aligned} \text{Or } B &= \frac{B_H}{\cos \delta} = \frac{0.3 \times 10^{-4}}{\cos 60^\circ} = \frac{0.3 \times 10^{-4}}{0.5} \\ &= 0.6 \times 10^{-4} \text{ T} \end{aligned}$$

3. At 52° of magnetic meridian, the apparent value of the horizontal component of earth's magnetic field,

$$B_H' = B_H \cos 52^\circ$$

Apparent angle of dip δ' ($= 45^\circ$) is given by

$$\tan \delta' = \frac{B_V}{B_H'}$$

$$\text{Or } \tan 45^\circ = \frac{B_V}{B_H \cos 52^\circ}$$

$$\text{Or } \frac{B_V}{B_H} = \tan 45^\circ \times \cos 52^\circ = 1 \times 0.6157 = 0.6157$$

$$\text{But } \frac{B_V}{B_H} = \tan \delta$$

where δ is the true angle of dip.

$$\text{Or } \delta = \tan^{-1} 0.6157$$

4. In first case: At the neutral point on axial line of a short magnet,

$$\frac{\mu_0}{4\pi} \cdot \frac{2M}{r^3} = B_H$$

$$\text{Or } \frac{10^{-7} \times 2 \times 5}{r^3} = 0.38 \times 10^{-4}$$

$$\text{Or } r^3 = 0.0263$$

$$\text{Or } r = 29.7 \times 10^{-2} \text{ m} = 29.7 \text{ cm}$$

In second case: At the neutral point on equatorial line of a short magnet,

$$\frac{\mu_0}{4\pi} \cdot \frac{M}{r^3} = B_H$$

$$\text{Or } \frac{10^{-7} \times 5}{r^3} = 0.38 \times 10^{-4}$$

$$\text{Or } r^3 = 0.0132$$

$$\text{Or } r = 23.6 \times 10^{-2} \text{ m} = 23.6 \text{ cm}$$

5. At the neutral point on equatorial line of a short magnet,

$$\frac{\mu_0}{4\pi} \cdot \frac{M}{r^3} = B_H$$

$$\text{Or } \frac{10^{-7} \times M}{(12.5 \times 10^{-2})^3} = 0.38 \times 10^{-4}$$

$$\text{Or } M = \frac{0.38 \times 10^{-4} \times (12.5 \times 10^{-2})^3}{10^{-7}} = 0.74$$

$$\begin{aligned} \text{(a) Now } B_{\text{axial}} &= \frac{\mu_0}{4\pi} \cdot \frac{2M}{r^3} = \frac{10^{-7} \times 2 \times 0.74}{(12.5 \times 10^{-2})^3} \\ &= 0.76 \times 10^{-4} \text{ T} = 0.76 \text{ G} \end{aligned}$$

Since at a point on the axial line, B_H and B_{axial} are in the same direction, total magnetic field,

$$B = B_{\text{axial}} + B_H = 0.76 + 0.38 = 1.14 \text{ G}$$

(b) When the magnet is turned through 180° , neutral points will be obtained on axial line.

Now, at the neutral point on axial line of a short magnet,

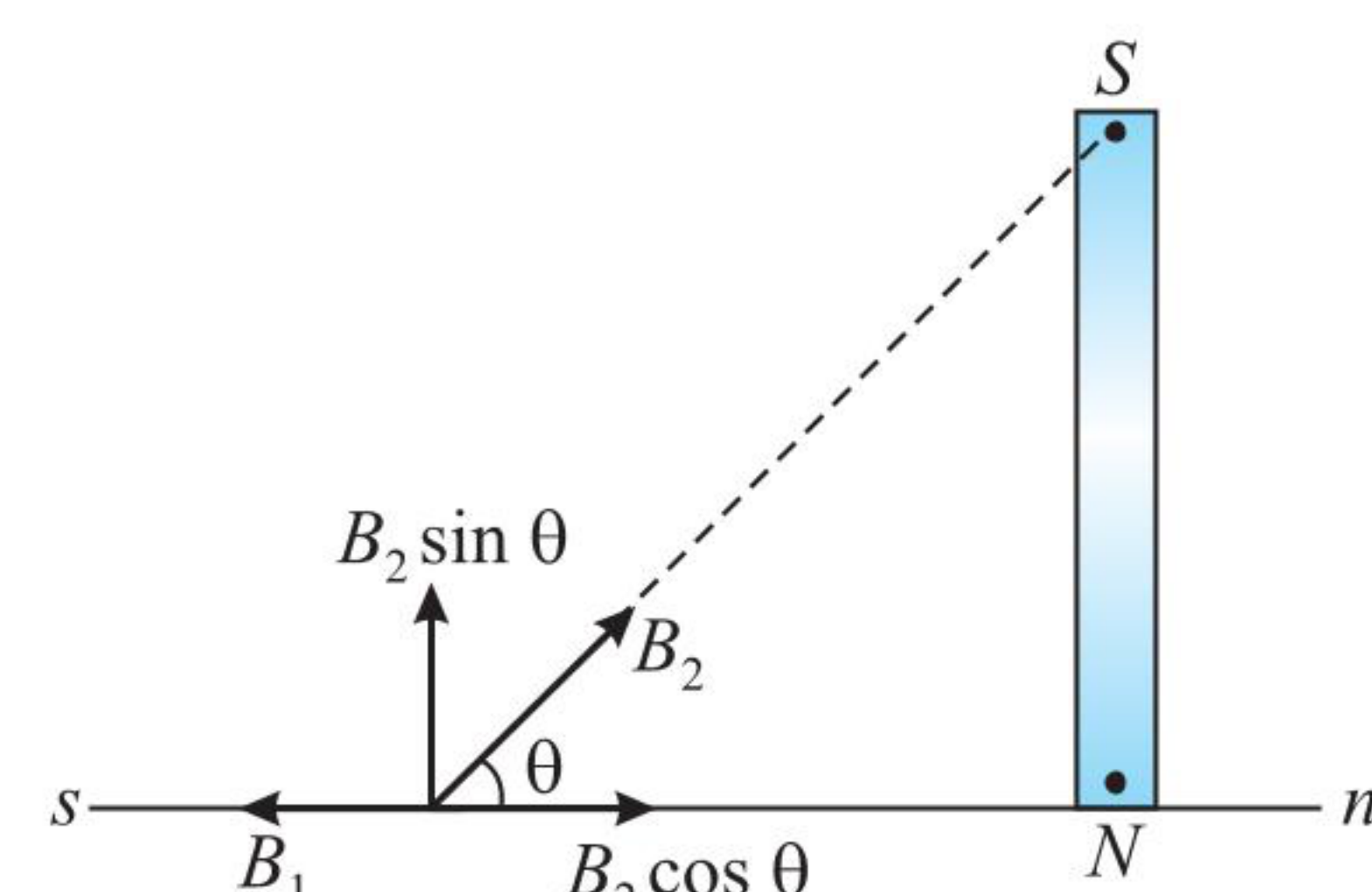
$$\frac{\mu_0}{4\pi} \cdot \frac{2M}{r^3} = B_H$$

$$\text{Or } \frac{10^{-7} \times 2 \times 0.74}{r^3} = 0.38 \times 10^{-4}$$

$$\text{Or } r^3 = 3.895 \times 10^{-3}$$

$$\text{Or } r = 0.157 \text{ m} = 15.7 \text{ cm}$$

6. The neutral point will be obtained at the point P_2 on the horizontal table as shown in figure.



At the neutral point, the horizontal component of earth's magnetic field (B_H) is balanced by the horizontal component of the magnetic field due to the magnet. Let m ($= 1 \text{ Am}$) be the pole strength of the magnet. Magnetic field due to N-pole of the magnet at the neutral point P_2 ,

$$B_1 = \frac{\mu_0}{4\pi} \cdot \frac{m}{(NP_2)^2} = \frac{10^{-7} \times 1}{(0.06)^2}$$

$$= 2.78 \times 10^{-5} \text{ T} \quad (\text{along } P_2S)$$

Magnetic field due to S-pole of the magnet at the neutral point P_2 ,

$$B_2 = \frac{\mu_0}{4\pi} \cdot \frac{m}{(SP_2)^2}$$

$$\text{Now, } SP_2 = \sqrt{(NS)^2 + (NP_2)^2}$$

$$= \sqrt{(0.08)^2 + (0.06)^2} = 0.1 \text{ m}$$

$$\therefore B_2 = \frac{10^{-7} \times 1}{(0.1)^2} = 10^{-5} \text{ T} \quad (\text{along } P_2S)$$

The magnetic field B_2 makes an angle θ with the horizontal table as shown in the figure.

Therefore, component of field B_2 along the horizontal

$$= B_2 \cos \theta = B_2 \times \frac{NP_2}{SP_2} = 10^{-5} \times \frac{0.06}{0.1}$$

$$= 0.6 \times 10^{-5} \text{ T} \quad (\text{along } P_2N)$$

Therefore, net horizontal component of the field of magnet

$$= B_1 - B_2 \cos \theta = 2.78 \times 10^{-5} - 0.6 \times 10^{-5}$$

$$= 2.18 \times 10^{-5} \text{ T} \quad (\text{along } P_2S)$$

At the neutral point,

$$B_H = B_1 - B_2 \cos \theta = 2.18 \times 10^{-5} \text{ T}$$

Exercise 3.3

1. If V is volume of the magnet, then $I = \frac{M}{V}$.

$$\text{Now, } V = \frac{\text{mass}}{\text{density}} = \frac{75 \times 10^{-3}}{7.5 \times 10^3} = 10^{-5} \text{ m}^3$$

$$\therefore I = \frac{2 \times 10^{-4}}{10^{-5}} = 20 \text{ Am}^{-1}$$

2. Here, $B = 1.0 \text{ Wb m}^{-2}$; $H = 2 \times 10^3 \text{ Am}^{-1}$

(a) Magnetic permeability,

$$\mu = \frac{B}{H} = \frac{1.0}{2 \times 10^3} = 5 \times 10^{-4} \text{ TmA}^{-1}$$

(b) Relative permeability,

$$\mu_r = \frac{\mu}{\mu_0} = \frac{5 \times 10^{-4}}{4\pi \times 10^{-7}} = 397.9$$

(c) Magnetic susceptibility,

$$\chi_m = \mu_r - 1 = 396.9 - 1 = 396.9$$

(d) Intensity of magnetization,

$$I = \chi_m H = 396.9 \times 2 \times 10^3 = 7.94 \times 10^5 \text{ Am}^{-1}$$

3. Here, $H = 1600 \text{ Am}^{-1}$; $\phi = 2.4 \times 10^{-5} \text{ Wb}$,

$$A = 0.2 \text{ cm}^2 = 0.2 \times 10^{-4} \text{ m}^2$$

Now, magnetic induction,

$$B = \frac{\phi}{A} = \frac{2.4 \times 10^{-5}}{0.2 \times 10^{-4}} = 1.2 \text{ T}$$

$$\text{Also, } \mu = \frac{B}{H} = \frac{1.2}{1600} = 7.5 \times 10^{-4} \text{ TmA}^{-1}$$

Therefore, relative permeability,

$$\mu_r = \frac{\mu}{\mu_0} = \frac{7.5 \times 10^{-4}}{4\pi \times 10^{-7}} = 596.8$$

Again, $B = \mu_0(H + I)$

$$\therefore I = \frac{B}{\mu_0} - H = \frac{1.2}{4\pi \times 10^{-7}} - 1600$$

$$= 9.55 \times 10^5 - 1600 = 9.534 \times 10^5 \text{ Am}^{-1}$$

$$\text{Finally, } \chi_m = \mu_r - 1 = 596.8 - 1 = 595.8$$

4. Mean radius of toroid,

$$r = \frac{11+12}{2} = 11.5 \text{ cm} = 11.5 \times 10^{-2} \text{ m}$$

Therefore, number of turns per unit length of the toroid,

$$n = \frac{\text{total number of turns}}{2\pi r} = \frac{3000}{2\pi \times 11.5 \times 10^{-2}}$$

$$= 4.152 \times 10^3 \text{ m}^{-1}$$

Now, magnetic field in the core of solenoid,

$$B = \mu_0 \mu_r nI$$

$$\text{Or } \mu_r = \frac{B}{\mu_0 nI}$$

$$\text{Or } \mu_r = \frac{2.5}{4\pi \times 10^{-7} \times 4.152 \times 10^3 \times 0.70} = 684.5$$

5. Here, $n = 10^3 \text{ m}^{-1}$; $I = 2 \text{ A}$; $B = 1.5 \text{ Wb m}^{-2}$

(a) Magnetic induction due to the solenoid,

$$B = \mu_0 \mu_r nI$$

$$\text{Or } \mu_r = \frac{B}{\mu_0 nI}$$

$$\text{Or } \mu_r = \frac{1.5}{4\pi \times 10^{-7} \times 10^3 \times 2} = 596.8$$

$$(b) \chi_m = \mu_r - 1 = 596.8 - 1 = 595.8$$

$$(c) H = \frac{B}{\mu} = \frac{B}{\mu_0 \mu_r} = \frac{1.5}{4\pi \times 10^{-7} \times 596.8}$$

$$= 2000 \text{ Am}^{-1}$$

$$(d) I = \chi_m H = 595.8 \times 2000 = 1.192 \times 10^6 \text{ Am}^{-1}$$

6. According to Curie's law

$$\chi_m = \frac{C}{T}$$

$$\therefore \frac{\chi_m}{\chi_m'} = \frac{T'}{T}$$

$$\text{Or } T' = T \times \frac{\chi_m}{\chi_m'} = \frac{300 \times 1.2 \times 10^{-5}}{1.44 \times 10^{-5}} = 250 \text{ K}$$

7. Here, hysteresis loss per unit volume per cycle

$$= 300 \text{ Jm}^{-3} \text{ cycle}^{-1}$$

Time, $t = 1 \text{ hour} = 3600 \text{ s}$, frequency, $f = 50 \text{ cycles s}^{-1}$

Therefore, number of hysteresis cycle in 1 hour,

$$N = f \times t = 50 \times 3600 = 1.8 \times 10^5$$

Volume of the iron specimen,

$$V = \frac{\text{mass}}{\text{density}} = \frac{12}{7500} = 1.60 \times 10^{-3} \text{ m}^3$$

Hence, loss of energy in one hour,

$$W = \text{hysteresis loss per unit volume per cycle} \times V \times N \\ = 300 \times 1.6 \times 10^{-3} \times 1.8 \times 10^5 = 8.64 \times 10^4 \text{ J}$$

8. (a) Here, number of atoms per unit volume in iron,

$$n = 8.52 \times 10^{28} \text{ m}^{-3}$$

Magnetic moment of each iron atom $= 2\mu_B$

$$= 2 \times 9.27 \times 10^{-24} = 1.854 \times 10^{-23} \text{ Am}^2$$

Now, intensity of magnetization is equal to magnetic moment per unit volume.

Therefore, the maximum value of magnetization,

$$I_{\text{max}} = n \times 2\mu_B = 8.52 \times 10^{28} \times 1.854 \times 10^{-23} \\ = 1.58 \times 10^6 \text{ Am}^{-1}$$

(b) Now, $B = \mu_0(H + I)$

Since no magnetic field is applied, $H = 0$

$$\therefore B = \mu_0 I = 4\pi \times 10^{-7} \times 1.58 \times 10^6 = 1.985 \text{ T}$$

$$B_2 = \frac{\mu_0}{4\pi} \frac{2M \times 20}{(20^2 - l^2)^2}$$

$$\frac{B_1}{B_2} = \frac{10}{(10^2 - l^2)^2} \times \frac{(20^2 - l^2)^2}{20}$$

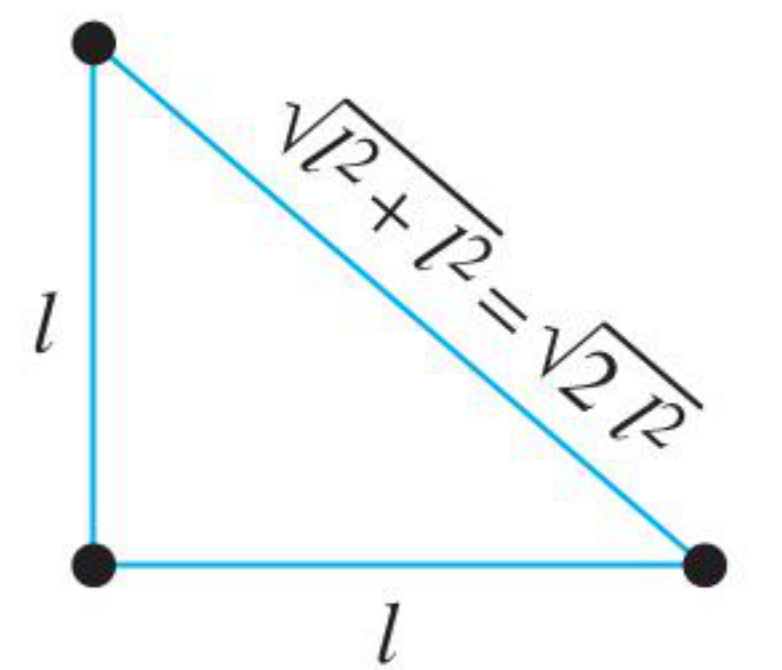
$$\text{or } \frac{125}{10} = \frac{(20^2 - l^2)^2}{2(10^2 - l^2)^2}, \text{ on solving we get}$$

$$\text{or } l = 5 \text{ cm}$$

$$\text{Length} = 2l = 10 \text{ cm.}$$

10. (3) Here, distance between two poles $= \sqrt{2}l$

$$\text{Hence, } M = m \times \sqrt{2}l = \sqrt{2}ml$$



$$11. (4) W = mB [\cos 0^\circ - \cos 60^\circ] = \frac{mB}{2}$$

$$\tau = mB \sin 60^\circ, \tau = mB \times \frac{\sqrt{3}}{2} = W \times \sqrt{3}$$

$$12. (2) \tau = MH \sin \theta$$

$$\therefore \sin \theta = \frac{\tau}{MH} = \frac{1}{2} \quad \therefore \theta = 30^\circ$$

$$13. (1) \tau = MB \sin 90^\circ = 1.42 \text{ N-m}$$

$$14. (4) F = \frac{\mu_0}{4\pi} \cdot \frac{6M_1M_2}{r^4}$$

$$\text{i.e., } \propto \frac{1}{r^4} \quad \therefore F' = \frac{F}{16} = \frac{8}{16} = 0.5 \text{ N}$$

$$15. (4) W = MB (\cos \theta_1 - \cos \theta_2)$$

When the magnet is rotated from 0° to 60° , then work done is 0.8 J.

$$0.8 = MB (\cos 0^\circ - \cos 60^\circ) = \frac{MB}{2}$$

$$MB = 1.6 \text{ N-m}$$

In order to rotate the magnet through an angle of 30° , i.e., from 60° to 90° , the work done is

$$W' = MB (\cos 60^\circ - \cos 90^\circ) \\ = MB \left(\frac{1}{2} - 0 \right) = \frac{MB}{2} = \frac{1.6}{2} = 0.8 \text{ J} = 0.8 \times 10^7 \text{ ergs}$$

$$16. (2) 2 = 2\pi \sqrt{\frac{I}{MH}}$$

$$1 = 2\pi \sqrt{\frac{I}{M(F-H)}}$$

$$\text{Dividing, } \frac{2}{1} = \sqrt{\frac{F-H}{H}} \Rightarrow \frac{H}{F} = \frac{1}{3}$$

$$17. (1) v = \frac{1}{2\pi} \sqrt{\frac{MB_H}{I}}$$

$$v' = \frac{1}{2\pi} \sqrt{\frac{M(B_H + B)}{I}}$$

$$v'' = \frac{1}{2\pi} \sqrt{\frac{M(B_H - B)}{I}}$$

$$\left(\frac{v'}{v} \right) = \frac{B_H + B}{B_H} = \frac{14 \times 14}{10 \times 10} = \frac{B_H + B}{B_H}$$

Exercises

Single Correct Answer Type

- (3) Pole strength becomes half due to change in breadth. Change in length does not affect pole strength.
- (4) New magnetic moment,

$$M' = \frac{2M \sin(\theta/2)}{\theta} = \frac{2M \sin(\pi/6)}{\pi/3} = \frac{2M \times \frac{1}{2}}{\pi/3} = \frac{3M}{\pi}$$

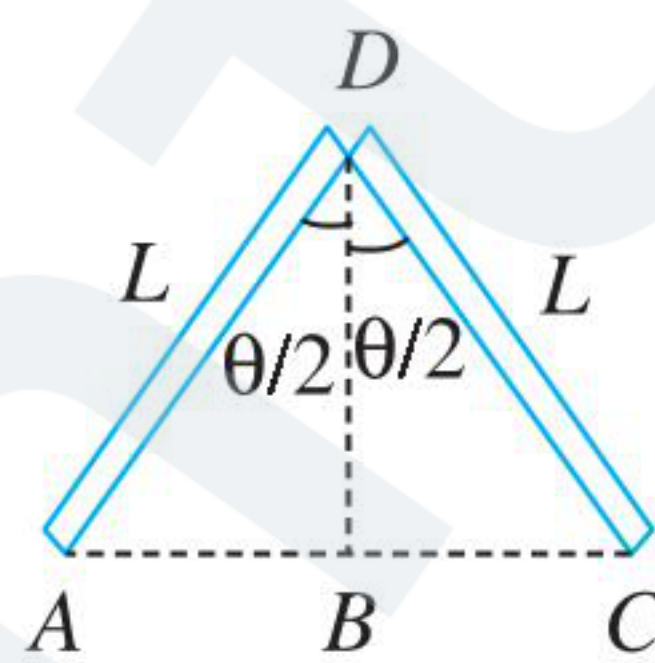
- (4) When the magnet is bent, the length of the magnet is:

$$AC = AB + BC$$

$$= L \sin \frac{\theta}{2} + L \sin \frac{\theta}{2}$$

$$= 2L \sin(\theta/2) = 2L \sin(60^\circ/2)$$

$$= 2L \sin 30^\circ = L$$



- (1) All the magnitude moment vectors can be represented both in magnitude and direction by the three sides of a triangle taken in the same order.
- (3) The hysteresis loss for soft iron is low. On the other hand, hysteresis loss for steel is high.

$$6. (2) \tau = MB \sin \theta$$

τ and B are constants.

$$\text{Now, } \sqrt{3} \sin \theta = 1 \times \sin(90^\circ - \theta)$$

$$\text{or } \tan \theta = \frac{1}{\sqrt{3}} \quad \text{or } \theta = 30^\circ.$$

$$7. (2) \tau = MB \sin \theta$$

$$\tau = \tau_0 \sin \theta$$

$$\frac{1}{2} \tau_0 = \tau_0 \sin \theta \quad \text{or } \sin \theta = \frac{1}{2} \quad \text{or } \theta = 30^\circ$$

$$8. (3) \tau = (m \times 2l) B \sin \theta$$

$$= 48 \times 25 \times 10^{-2} \times 0.15 \times \sin 30^\circ \text{ N-m}$$

$$= 0.9 \text{ N-m.}$$

$$9. (3) B_1 = \frac{\mu_0}{4\pi} \frac{2M \times 10}{(10^2 - l^2)^2}$$

$$\text{or } 1.96 B_H = B_H + B \quad \text{or } 0.96 B_H = B$$

$$\text{Now } \left(\frac{v''}{v}\right)^2 = \frac{B_H - B}{B_H} \quad \text{or } \frac{(v'')^2}{100} = \frac{B_H - 0.96 B_H}{B_H}$$

$$\text{or } (v'')^2 = 4 \quad \text{or } v'' = 2$$

$$18. (1) \tau = MB \sin \theta$$

$$\text{or } \sin \theta = \frac{\tau}{MB} = \frac{1.2 \times 10^{-3}}{60 \times 40 \times 10^{-6}} = 0.5$$

$$\text{or } \theta = 30^\circ$$

$$19. (2) T = 2\pi \sqrt{\frac{I_1 + I_2}{(2M + M)H}} = 3 \quad \dots(i)$$

$$T' = 2\pi \sqrt{\frac{I_1 + I_2}{(2M - M)H}} \quad \dots(ii)$$

$$\frac{T'}{T} = \sqrt{3} \quad \text{or } T' = 3\sqrt{3} \text{ s}$$

20. (3) In the first case, it is vibrating in the total field of earth $n \propto \sqrt{B}$. In the second case, it is vibrating in vertical component only as the horizontal component is zero.

$$\therefore n' \propto \sqrt{V} \quad \text{but } V < B \quad \therefore n' < n.$$

$$21. (1) n \propto \sqrt{H} \quad \text{or } \frac{9}{16} = \frac{H}{35 \times 10^{-5}}$$

$$\text{or } H = \frac{31.5 \times 10^{-5} \times 9}{16} = 1.98 \times 10^{-5} \text{ T}$$

$$22. (2) \text{ Here, } \frac{M_1}{M_2} = \frac{T_2^2 + T_1^2}{T_2^2 - T_1^2} = \frac{n_1^2 + n_2^2}{n_1^2 - n_2^2} = \frac{400 + 225}{400 - 225} = \frac{625}{175} = \frac{25}{7}$$

23. (2) Since $T \propto \sqrt{I}$ and $I \propto m$, when mass is made 4 times, the time period becomes $2T$.

$$24. (1) T = 2\pi \sqrt{\frac{I}{MB}}$$

$$T_1 = 2\pi \sqrt{\frac{I_1}{M_1 B}} \quad \left(\because I_1 = \frac{I}{4} \text{ and } M_1 = \frac{M}{2} \right)$$

$$\therefore \frac{T_1}{T_2} = \frac{1}{\sqrt{2}} \Rightarrow T_1 = \frac{T_0}{\sqrt{2}}$$

$$25. (2) T = 2\pi \sqrt{\frac{I}{MB \cos \delta}}$$

$$v = \frac{1}{2\pi} \sqrt{\frac{MB \cos \delta}{I}}$$

$$v = \sqrt{B \cos \delta}$$

$$\text{or } B \propto \frac{v^2}{\cos \delta}$$

$$\frac{B_1}{B_2} = \frac{400}{\cos 30^\circ} \times \frac{\cos 60^\circ}{225} = \frac{16 \times 2}{9 \times \sqrt{3}} \times \frac{1}{2} = 16 : 9\sqrt{3}$$

26. (1) Theory based

27. (4) $B_0 = V_0$ also total intensity

$$B = \sqrt{B_0^2 + V_0^2} \Rightarrow B = \sqrt{2} B_0$$

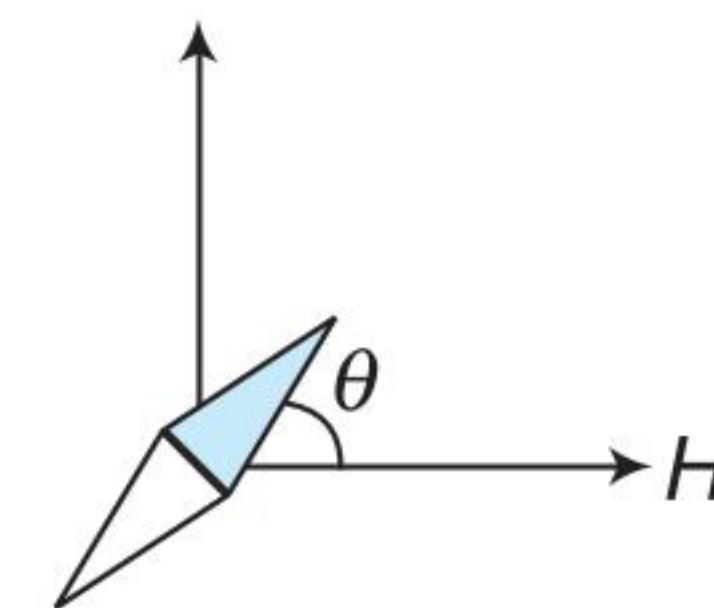
$$28. (1) \text{ Here, } \tan \delta = \frac{V}{H} \text{ and } \tan \delta' = \frac{V}{H \cos \theta}$$

$$\therefore \frac{\tan \delta'}{\tan \delta} = \frac{V}{H \cos \theta} \times \frac{H}{V} = \frac{1}{\cos \theta}$$

29. (1) In given case, H and H_0 are perpendicular to each other.

$$\text{From figure } \tan \theta = \frac{H_0}{H}$$

$$\Rightarrow \theta = \tan^{-1} \left(\frac{H_0}{H} \right)$$



30. (1) Let the real dip be ϕ , then $\tan \phi = \frac{B_V}{B_H}$

For apparent dip,

$$\tan \phi' = \frac{B_V}{B_H \cos \beta} = \frac{B_V}{B_H \cos 30^\circ} = \frac{2B_V}{\sqrt{3}B_H}$$

$$\text{or } \tan 45^\circ = \frac{2}{\sqrt{3}} \cdot \tan \phi \quad \text{or } \phi = \tan^{-1} \left(\frac{\sqrt{3}}{2} \right)$$

31. (2) For a diamagnetic substance χ is small, negative and independent of temperature.

32. (1) Susceptibility of a paramagnetic substance is independent of magnetising field.

33. (1) Susceptibility of a ferromagnetic substance falls with rise of temperature $\left(\chi = \frac{C}{T - T_c} \right)$ and the substance becomes paramagnetic above Curie temperature, so magnetic susceptibility becomes very small above Curie temperature.

$$34. (1) X = C \times \frac{1}{T} = \frac{0.4}{7 \times 10^{-3}} = 57 \text{ K}$$

35. (2) With rise in temperature their magnetic susceptibility decreases i.e.

$$\chi_m \propto \frac{1}{T}$$

36. (3) Diamagnetic substances are repelled by magnetic field.

37. (3) If the temperature is decreased, the thermal vibrations will be reduced. So, there would not be negative effect on magnetisation.

38. (2) By Curie's law, $\chi_m \propto \frac{1}{T}$

$$0.5 T = K (273 + 27) \text{ or } T = 600 \text{ K}$$

$$\therefore t = (600 - 273)^\circ \text{C} = 327^\circ \text{C}$$

39. (2) Steel of an alloy Alnico (Al + Ni + Co) is used for making permanent magnets. Hysteresis loss is high. Moreover, coercivity is high and retentivity is low. Soft iron is used for electromagnets. Retentivity is high. Coercivity is low.

40. (4) $M_1 B \sin 30^\circ \propto (180^\circ - 30^\circ)$
and $M_2 B \sin 30^\circ \propto (270^\circ - 30^\circ)$

$$\therefore \frac{M_1}{M_2} = \frac{150^\circ}{240^\circ} = \frac{5}{8}$$

41. (3) In equilibrium, $\tau_1 = \tau_2$

$$MB \sin \theta = \sqrt{3} MB \sin (90^\circ - \theta)$$

$$\therefore \tan \theta = \sqrt{3} \Rightarrow \theta = 60^\circ$$

42. (3) In the given figure, two magnets are perpendicular to each other.

$$B = \frac{\mu_0}{4\pi r^3} \sqrt{4+1} = \frac{\mu_0 M}{4\pi d^3} \sqrt{5}$$

$$43. (2) B = \frac{\mu_0 I}{2\pi r} = \frac{4\pi \times 10^{-7} \times 18}{2\pi \times 0.2} \text{ T} = 18 \mu\text{T}$$

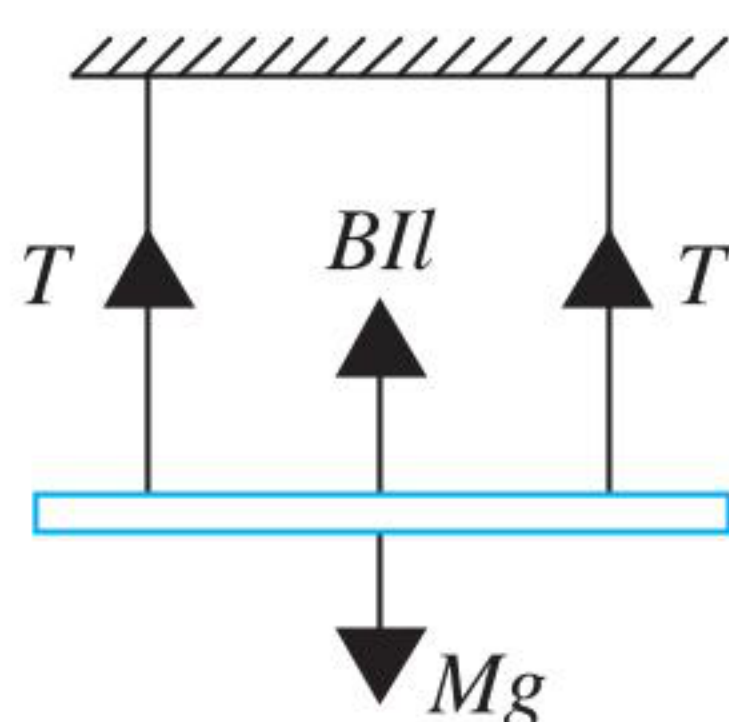
$$\text{Now, } T = 2\pi \sqrt{\frac{I}{MB_H}} \text{ and } T' = 2\pi \sqrt{\frac{I}{M(B_H - B)}}$$

$$\text{Dividing } \frac{T'}{T} = \sqrt{\frac{B_H}{B_H - B}} \text{ or } \frac{T'}{T} = \sqrt{\frac{24}{24 - 18}} = 2$$

$$T' = 2 \times 0.1 \text{ s} = 0.2 \text{ s}$$

$$44. (4) 2T + BIl = Mg$$

$$\text{or } T = \frac{Mg - BIl}{2}$$



$$45. (1) \text{ Here, } \tau = iAB \sin \theta; i = 0.1 \text{ A}$$

$$A = \frac{1}{2} \times \text{base} \times \text{height}$$

$$\text{or } A = \frac{1}{2} \times a \times \frac{a\sqrt{3}}{2} = \frac{\sqrt{3}a^2}{4} = \frac{\sqrt{3} \times (0.02)^2}{4}$$

$$= \sqrt{3} \times 10^{-4} \text{ m}^2; \theta = 90^\circ$$

$$\tau = 0.1 \times \sqrt{3} \times 10^{-4} \times 5 \times 10^{-2} \sin 90^\circ$$

$$= 5\sqrt{3} \times 10^{-7} \text{ N-m}$$

$$46. (2) \text{ As magnets are perpendicular to each other the resultant magnetic moment}$$

$$= \sqrt{M^2 + M^2} = \sqrt{2} M$$

$$\therefore T_1 = 2\pi \sqrt{\frac{2I}{\sqrt{2}MH}} \quad \dots(i)$$

$$\text{In the second case } T_2 = 2\pi \sqrt{\frac{I}{MH}}$$

$$\frac{T_2}{T_1} = \frac{1}{(2)^{1/4}}$$

$$\therefore T_2 = \frac{4}{(2)^{1/4}} = 3.36 \text{ s}$$

$$47. (2) \text{ Since } B \text{ and } H \text{ are } \perp \text{ to each other and the resultant field is inclined at an angle } 45^\circ \text{ with}$$

$$\text{So, } B = H$$

$$\frac{\mu_0}{4\pi} \frac{2M}{r^3} = H$$

$$\therefore r^3 = \frac{\mu_0}{4\pi} \frac{2M}{H} = 0.5 \text{ m}$$

$$48. (2) B_1 = 1.5 \times 10^{-2} \text{ T}$$

$$B_2 = ?$$

In equilibrium position,

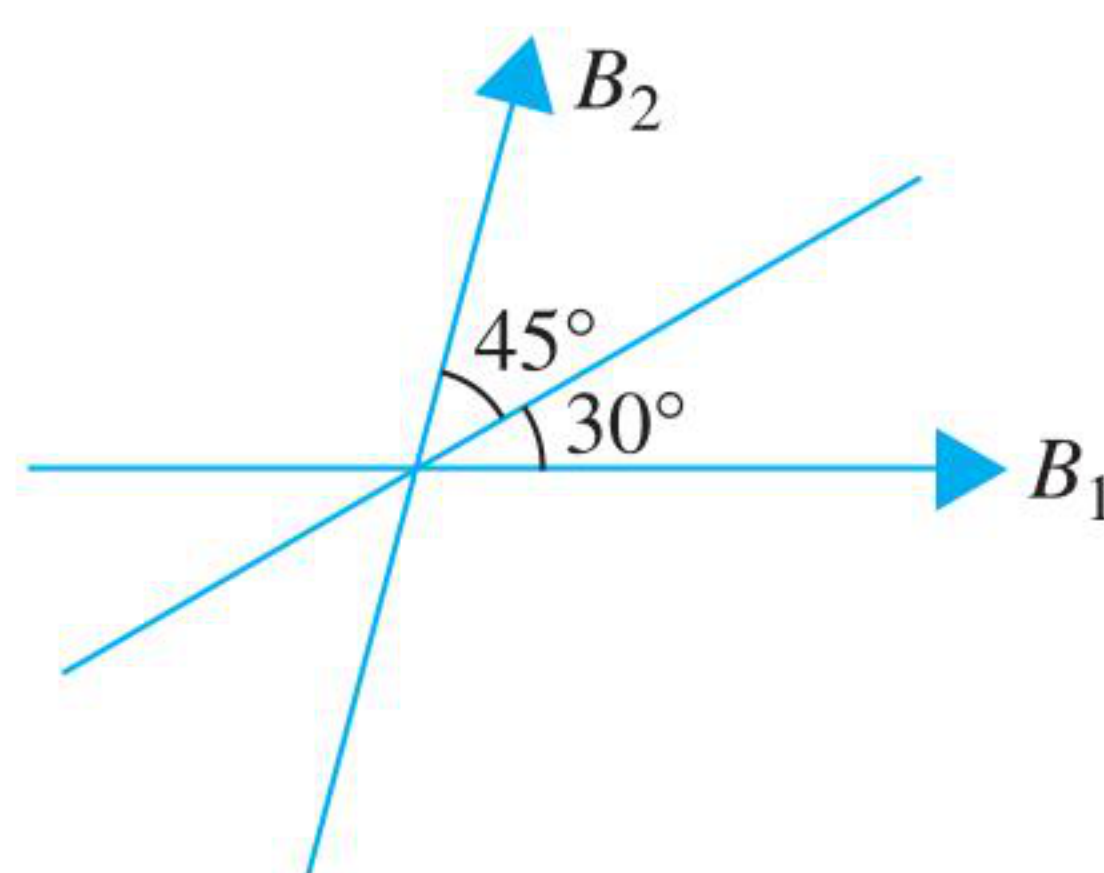
$$\tau_1 = \tau_2$$

$$MB_1 \sin \theta_1 = MB_2 \sin \theta_2$$

$$1.5 \times 10^{-21} \times \sin 30^\circ$$

$$= B_2 \times \sin 45^\circ$$

$$\text{Then } B = \frac{1.5}{\sqrt{2}} \times 10^{-2} \text{ T}$$



$$49. (2) \tan \delta = \frac{B_V}{B_H} \text{ and } \tan \delta_1 = \frac{B_V}{B_H \cos \theta}$$

$$\text{or } \tan \delta_1 = \frac{\tan \delta}{\cos \theta} = \tan \delta \sec \theta$$

$$50. (1) I = \frac{M}{V} = \frac{\mu N}{V} = \frac{1.5 \times 10^{-23} \times 2 \times 10^{26}}{1} = 3 \times 10^3 \text{ A/m}$$

$$51. (3) \text{ The number of atoms per unit volume in a specimen,}$$

$$n = \frac{\rho N_A}{A}$$

$$\text{For iron, } \rho = 7.8 \times 10^3 \text{ kg m}^{-3},$$

$$N_A = 6.02 \times 10^{26} / \text{kg mol}, A = 56$$

$$\Rightarrow n = \frac{7.8 \times 10^3 \times 6.02 \times 10^{26}}{56} = 8.38 \times 10^{28} \text{ m}^{-3}$$

Total number of atoms in the bar is

$$N_0 = nV = 8.38 \times 10^{28} \times (5 \times 10^{-2} \times 1 \times 10^{-2} \times 1 \times 10^{-2})$$

$$N_0 = 4.19 \times 10^{23}$$

The saturated magnetic moment of bar

$$= 4.19 \times 10^{23} \times 1.8 \times 10^{-23} = 7.54 \text{ Am}^2$$

$$52. (4) \text{ We have, } B = \mu_0 H + \mu_0 I$$

$$\text{or } I = \frac{B - \mu_0 H}{\mu_0} \text{ or } I = \frac{\mu H - \mu_0 H}{\mu_0} = \left(\frac{\mu}{\mu_0} - 1 \right) H$$

$$I = (\mu_r - 1)H$$

For a solenoid of n -turns per unit length and current i

$$H = ni$$

$$\therefore I = (\mu_r - 1)ni = (1000 - 1) \times 500 \times 0.5$$

$$I = 2.5 \times 10^5 \text{ Am}^{-1}$$

$$\therefore \text{Magnetic moment } M = IV$$

$$M = 2.5 \times 10^5 \times 10^{-4} = 25 \text{ Am}^2$$

$$53. (4) \text{ The bar magnet coercively } 4 \times 10^3 \text{ Am}^{-1} \text{ i.e., it requires a magnetic intensity } H = 4 \times 10^3 \text{ Am}^{-1} \text{ to get demagnetized. Let } i \text{ be the current carried by solenoid having } n \text{ number of turns per metre length, then by definition } H = ni. \text{ Here } H = 4 \times 10^3 \text{ Amp turn metre}^{-1}$$

$$n = \frac{N}{l} = \frac{60}{0.12} = 500 \text{ turn metre}^{-1}$$

$$\Rightarrow i = \frac{H}{n} = \frac{4 \times 10^3}{500} = 8.0 \text{ A}$$

$$54. (2) I = \frac{M}{V} = \frac{M}{\text{mass/density}}, \text{ given mass} = 1 \text{ gm} = 10^{-3} \text{ kg,}$$

$$\text{and density} = 5 \text{ gm/cm}^3 = \frac{5 \times 10^{-3} \text{ kg}}{(10^{-2})^3 \text{ m}^3} = 5 \times 10^3 \text{ kg/m}^3$$

$$\text{Hence } I = \frac{6 \times 10^{-7} \times 5 \times 10^3}{10^{-3}} = 3 \text{ A/m}$$

$$55. (1) E = nAVt = nA \frac{m}{d} t = \frac{50 \times 250 \times 10 \times 3600}{7.5 \times 10^3} = 6 \times 10^4 \text{ J}$$

$$56. (2) T = 2\pi \sqrt{\frac{I}{MB_H}} \Rightarrow T \propto \frac{1}{\sqrt{M}} \Rightarrow \frac{T_1}{T_2} = \sqrt{\frac{M_2}{M_1}}$$

$$\text{If } M_1 = 100 \text{ then } M_2 = (100 - 36) = 64$$

$$\text{So } \frac{T_1}{T_2} = \sqrt{\frac{64}{100}} = \frac{8}{10} \Rightarrow T_2 = \frac{10}{8} T_1 = 1.25 T_1$$

So % increase in time period = 25%

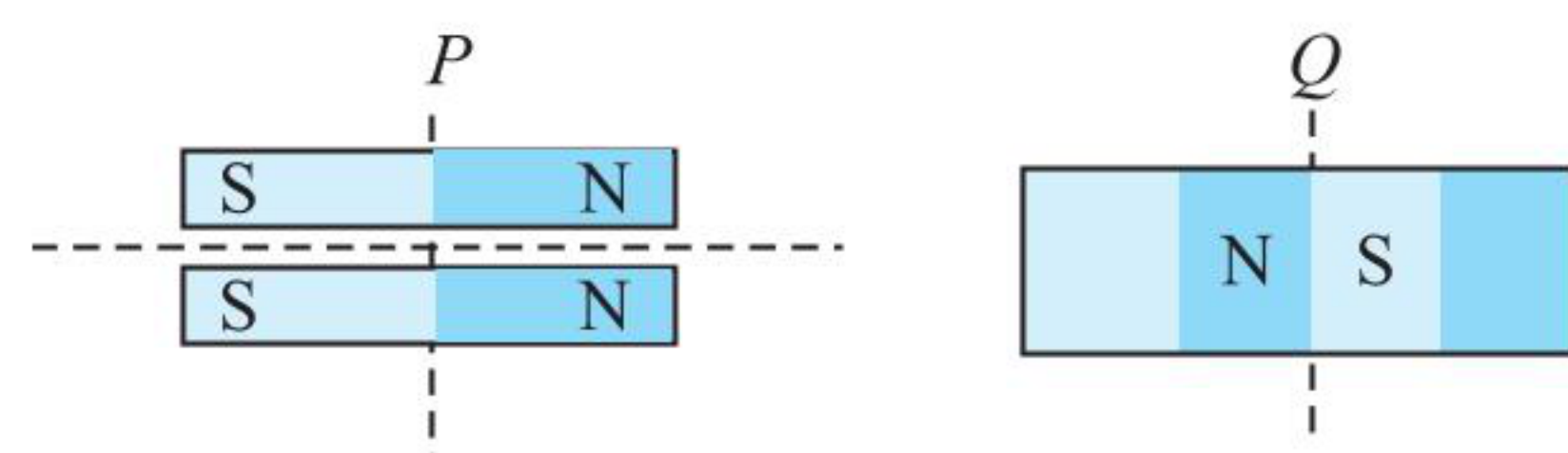
57. (1) $W = MB(\cos\theta_1 - \cos\theta_2) = MB(\cos 0^\circ - \cos 60^\circ)$

$$= MB\left(1 - \frac{1}{2}\right) = \frac{MB}{2}$$

and $\tau = MB \sin\theta = MB \sin 60^\circ = MB \frac{\sqrt{3}}{2}$

$$\therefore \tau = \left(\frac{MB}{2}\right)\sqrt{3} \Rightarrow \tau = \sqrt{3}W$$

58. (3) If pole strength, magnetic moment and length of each part are m' , M' and L' respectively then



$$\Rightarrow m' = \frac{m}{2}$$

$$\Rightarrow m' = m$$

$$\Rightarrow L' = L$$

$$\Rightarrow L' = \frac{L}{2} \Rightarrow M' = \frac{M}{2} \Rightarrow M' = \frac{M}{2}$$

59. (3) $\tau = MB \sin\theta \Rightarrow \tau \propto \sin\theta$

$$\Rightarrow \frac{\tau_1}{\tau_2} = \frac{\sin\theta_1}{\sin\theta_2} \Rightarrow \frac{\tau}{\tau/2} = \frac{\sin 90^\circ}{\sin\theta_2}$$

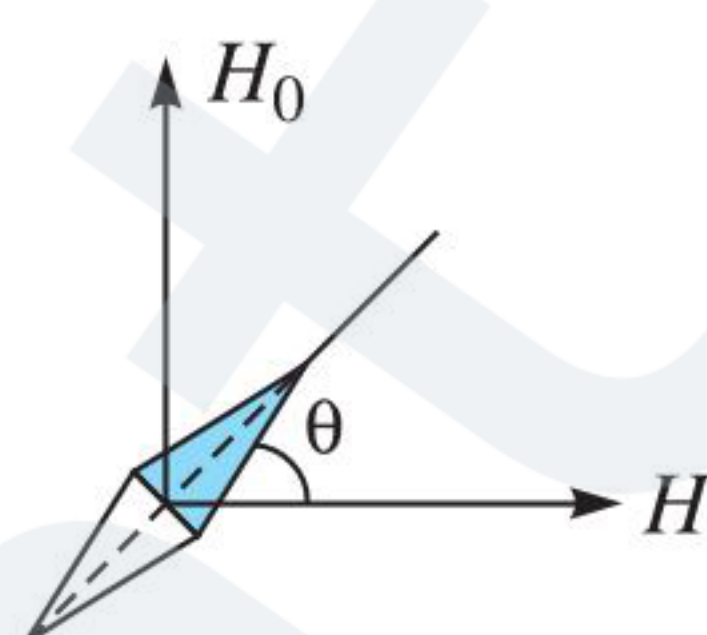
$$\Rightarrow \sin\theta_2 = \frac{1}{2} \Rightarrow \theta_2 = 30^\circ$$

$$\Rightarrow \text{Angle of rotation} = 90^\circ - 30^\circ = 60^\circ$$

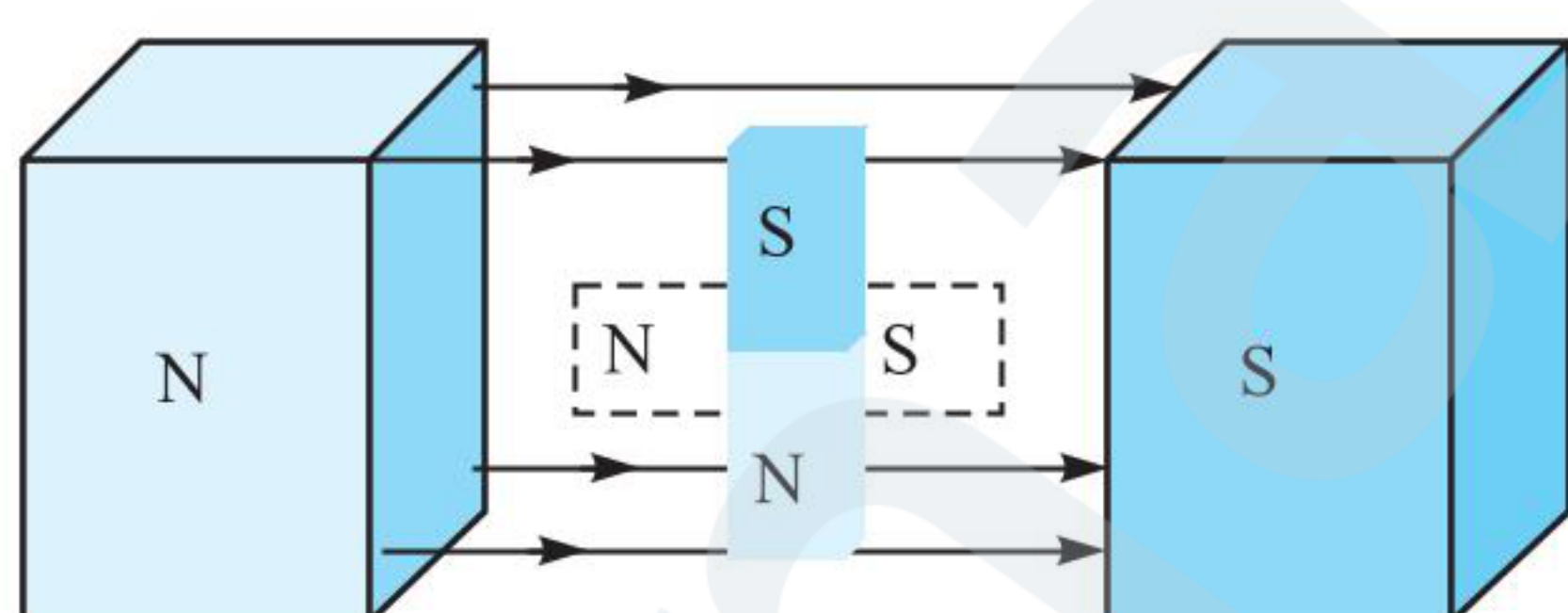
60. (1) In given case H and H_0 are perpendicular to each other.

From figure $\tan\theta = \frac{H_0}{H}$

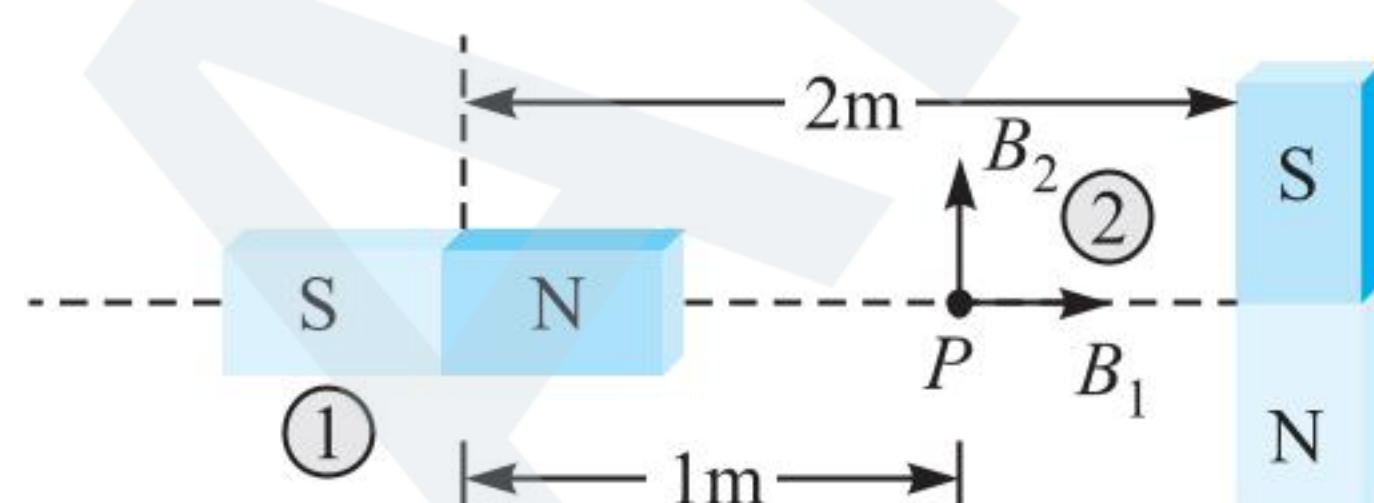
$$\Rightarrow \theta = \tan^{-1}\left(\frac{H_0}{H}\right)$$



61. (1) A diamagnetic rod set itself perpendicular to the field if free to rotate between the poles of a magnet as in this situation the field is strongest near the poles.



62. (2) With respect to 1st magnet, P lies in end side-on position



$$\therefore B_1 = \frac{\mu_0}{4\pi} \left(\frac{2M}{d^3} \right) \quad (\text{RHS})$$

With respect to 2nd magnet, P lies in broad sideon position.

$$\therefore B_2 = \frac{\mu_0}{4\pi} \left(\frac{M}{d^3} \right) \quad (\text{Upward})$$

$$B_1 = 10^{-7} \times \frac{2 \times 1}{1} = 2 \times 10^{-7} \text{ T}, B_2 = \frac{B_1}{2} = 10^{-7} \text{ T}$$

As B_1 and B_2 are mutually perpendicular, hence the resultant magnetic field

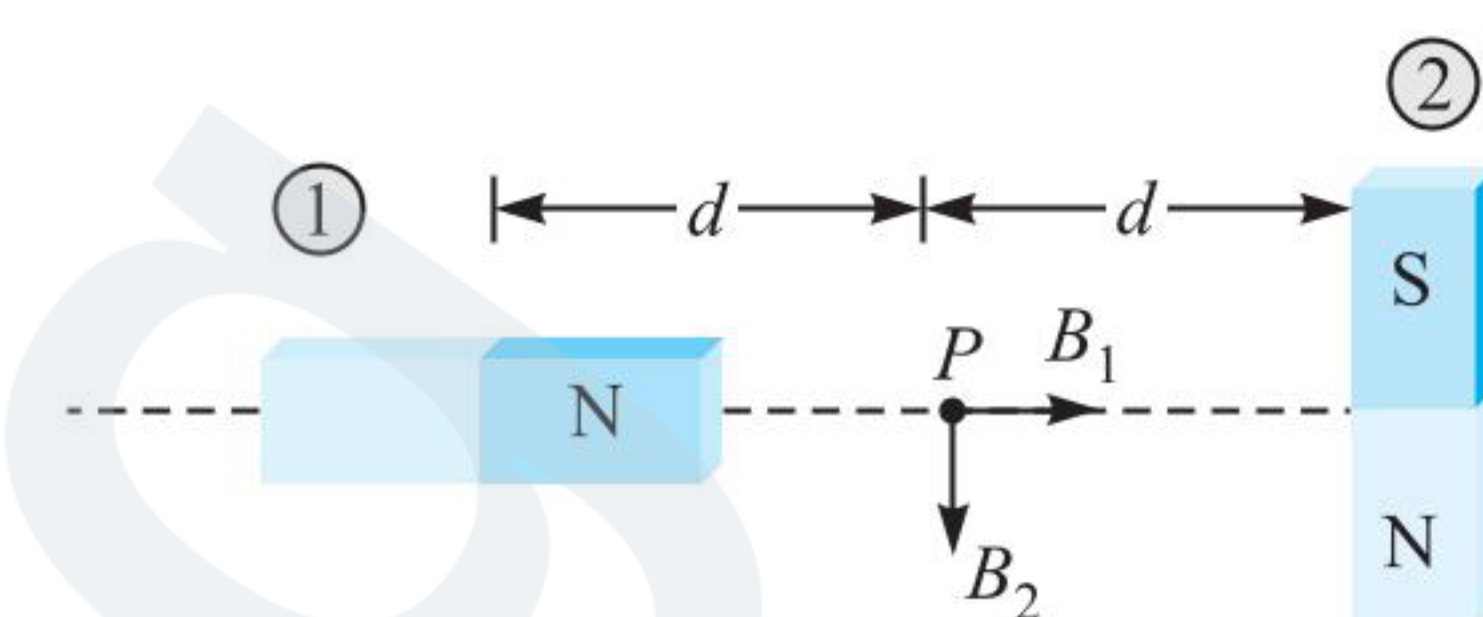
$$B_R = \sqrt{B_1^2 + B_2^2} = \sqrt{(2 \times 10^{-7})^2 + (10^{-7})^2} = \sqrt{5} \times 10^{-7} \text{ T}$$

63. (4) At point P net magnetic field $B_{\text{net}} = \sqrt{B_1^2 + B_2^2}$

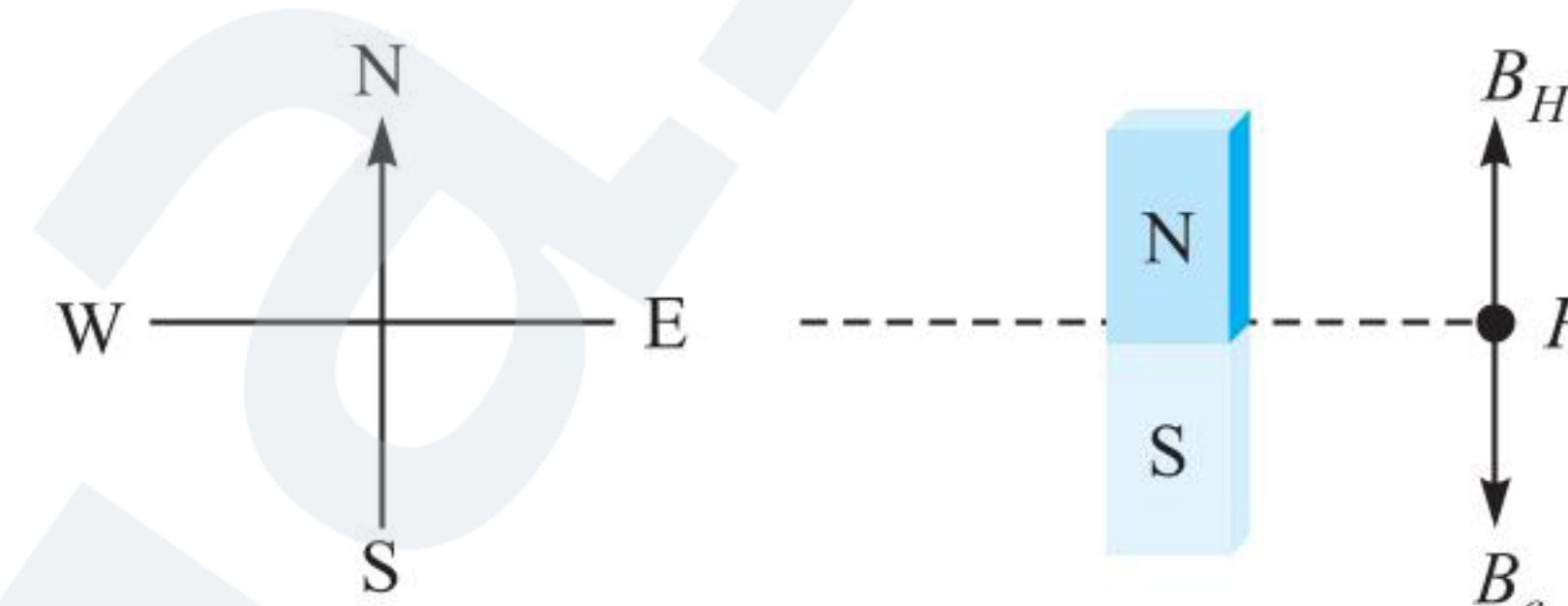
$$\text{where } B_1 = \frac{\mu_0}{4\pi} \cdot \frac{2M}{d^3}$$

$$\text{and } B_2 = \frac{\mu_0}{4\pi} \cdot \frac{M}{d^3}$$

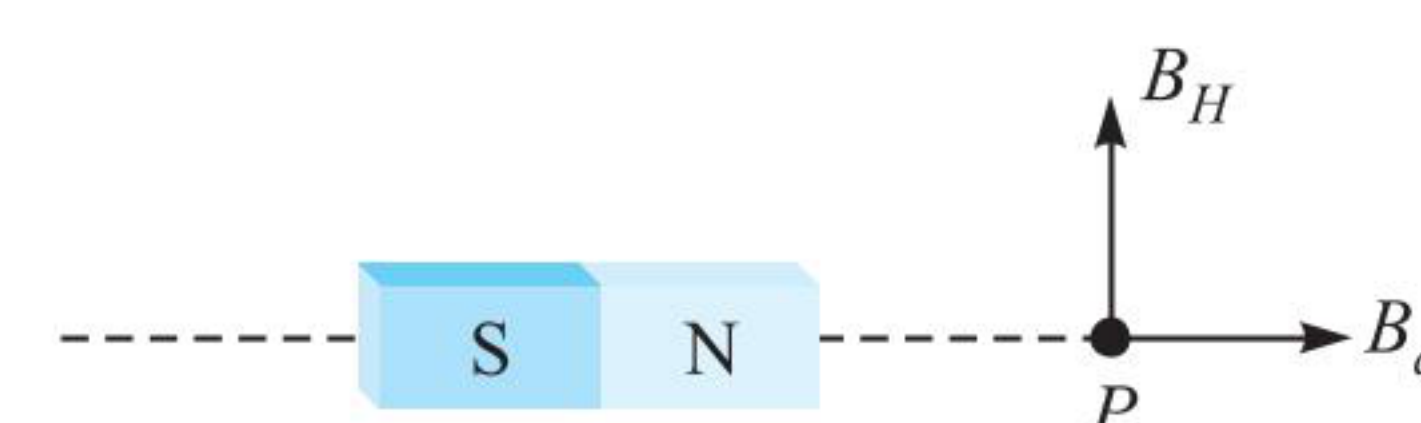
$$\Rightarrow B_{\text{net}} = \frac{\mu_0}{4\pi} \cdot \frac{\sqrt{5}M}{d^3}$$



64. (4) Initially neutral point obtained on equatorial line and at neutral point $|B_H| = |B_e|$



where B_H = Horizontal component of earth's magnetic field, B_e = Magnetic field due to bar magnet on its equatorial line.

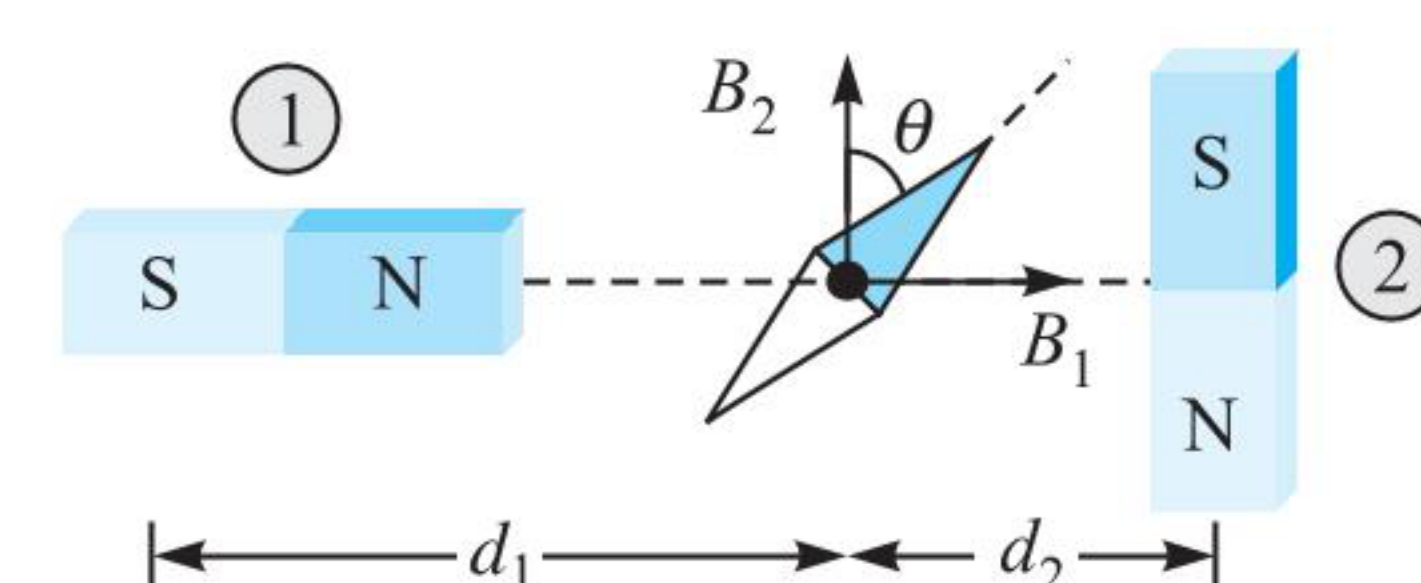


Finally, point P comes on axial line of the magnet and at P , net magnetic field

$$B = \sqrt{B_a^2 + B_H^2}$$

$$= \sqrt{(2B_e)^2 + (B_H)^2} = \sqrt{(2B_H)^2 + B_H^2} = \sqrt{5} B_H$$

65. (3) In equilibrium $B_1 = B_2 \tan\theta$



$$\Rightarrow \frac{\mu_0}{4\pi} \cdot \frac{2M}{d_1^3} = \frac{\mu_0}{4\pi} \cdot \frac{M}{d_2^3} \tan\theta$$

$$\Rightarrow \frac{d_1}{d_2} = (2 \cot\theta)^{1/3}$$

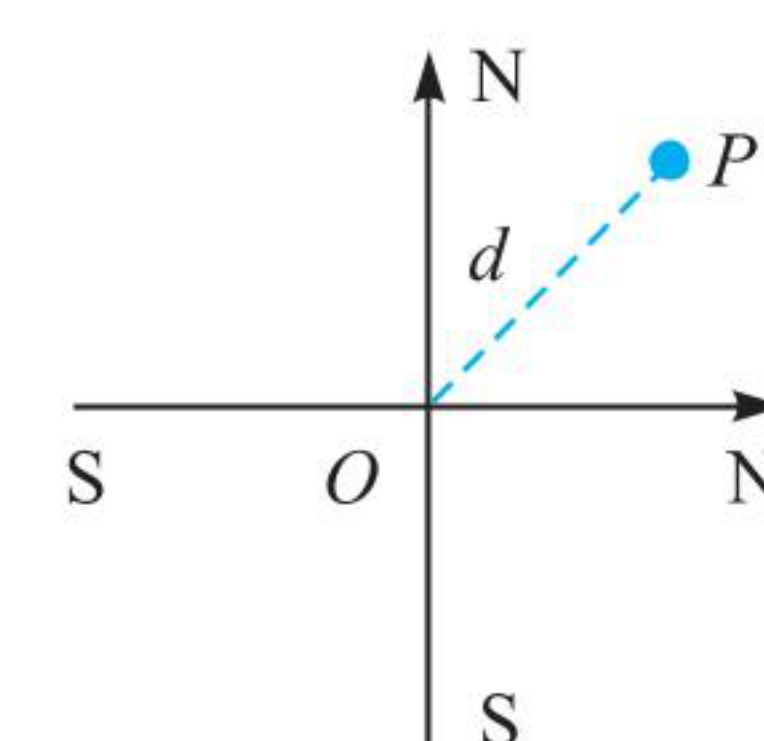
66. (3) Resultant magnetic moment of the two magnets is

$$M_{\text{net}} = \sqrt{M^2 + M^2} = \sqrt{2}M$$

Imagine a short magnet lying along OP with magnetic moment equal to $M\sqrt{2}$. Thus point P lies on the axial line of the magnet.

$\therefore$ Magnitude of magnetic field at P is given by

$$B = \frac{\mu_0}{4\pi} \cdot \frac{2\sqrt{2}M}{d^3}$$



67. (1) On passing current through the coil, it acts as a magnetic dipole. Torque acting on magnetic dipole is counter balanced by the moment of additional weight about position O .

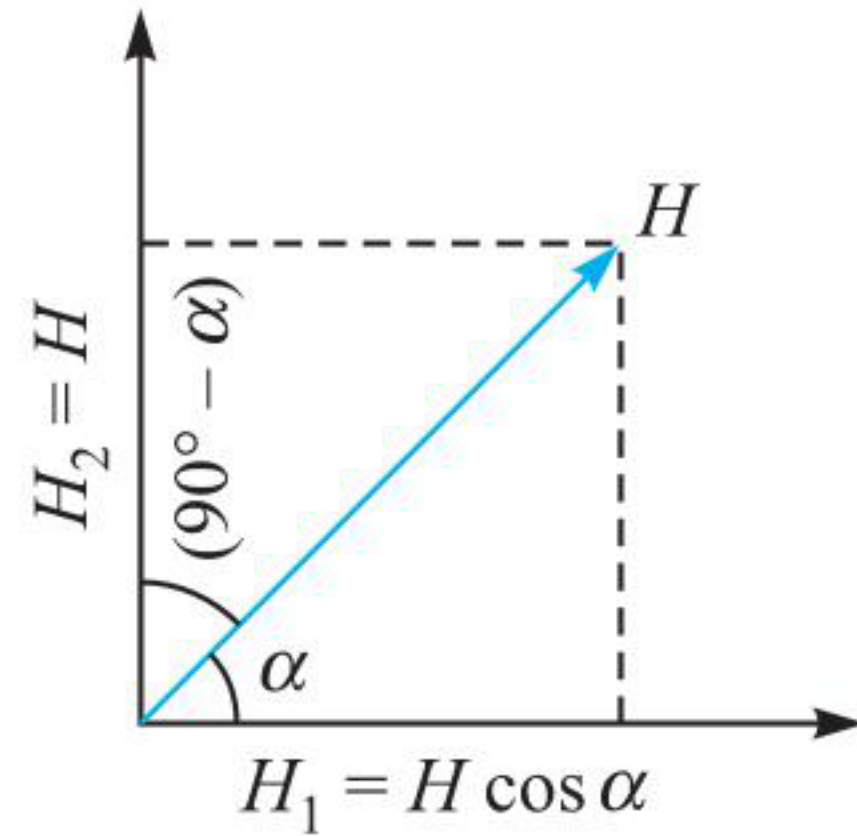
Torque acting on a magnetic dipole $\tau = MB \sin \theta = (NiA)B \sin 90^\circ = NiAB$.

Again $\tau = \text{force} \times \text{Lever arm} = \Delta mg \times l$

$$\Rightarrow NiAB = \Delta mgl$$

$$\Rightarrow B = \frac{\Delta mgl}{NiA} = \frac{60 \times 10^{-3} \times 9.8 \times 30 \times 10^{-2}}{200 \times 22 \times 10^{-3} \times 1 \times 10^{-4}} = 0.4 \text{ T}$$

68. (4) Let α be the angle which one of the planes make with the magnetic meridian the other plane makes an angle $(90^\circ - \alpha)$ with it. The components of H in these planes will be $H \cos \alpha$ and $H \sin \alpha$ respectively. If ϕ_1 and ϕ_2 are the apparent dips in these two planes, then



$$\tan \phi_1 = \frac{V}{H \cos \alpha} \text{ i.e. } \cos \alpha = \frac{V}{H \tan \phi_1} \quad \dots(i)$$

$$\tan \phi_2 = \frac{V}{H \sin \alpha} \text{ i.e., } \sin \alpha = \frac{V}{H \tan \phi_2} \quad \dots(ii)$$

Squaring and adding (i) and (ii), we get

$$\cos^2 \alpha + \sin^2 \alpha = \left(\frac{V}{H}\right)^2 \left(\frac{1}{\tan^2 \phi_1} + \frac{1}{\tan^2 \phi_2}\right)$$

$$\text{i.e. } 1 = \frac{V^2}{H^2} (\cot^2 \phi_1 + \cot^2 \phi_2)$$

$$\text{or } \frac{H^2}{V^2} = \cot^2 \phi_1 + \cot^2 \phi_2 \text{ i.e.}$$

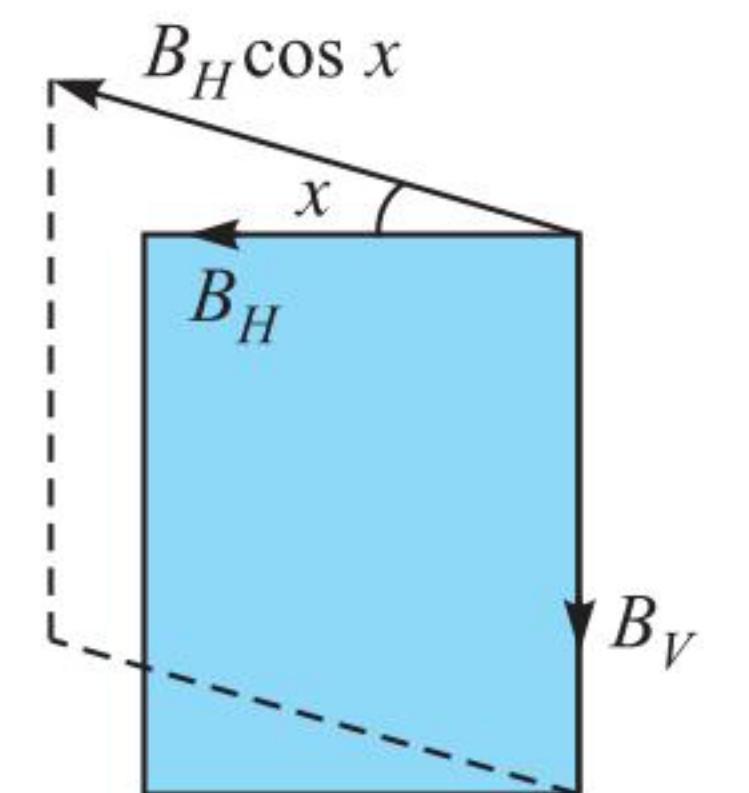
$$\cot^2 \phi = \cot^2 \phi_1 + \cot^2 \phi_2$$

This is the required result.

$$69. (1) \text{ In first case } \tan \theta = \frac{B_V}{B_H} \quad \dots(i)$$

$$\text{Second case } \tan \theta' = \frac{B_V}{B_H \cos x} \quad \dots(ii)$$

$$\text{From Eqs. (i) and (ii), } \frac{\tan \theta'}{\tan \theta} = \frac{1}{\cos x}$$



$$70. (3) \tan \theta = \frac{B_V}{B_H} \quad \dots(i)$$

If apparent dip is θ' then

$$\tan \theta' = \frac{B'_V}{B'_H} = \frac{B_V}{B_H \cos 30^\circ} = \frac{B_V}{B_H \times \frac{\sqrt{3}}{2}}$$

$$\Rightarrow \tan \theta' = \left(\frac{2}{\sqrt{3}}\right) \tan \theta \Rightarrow \tan \theta' > \tan \theta \Rightarrow \theta' > \theta$$

71. (2) In the given figure OQ refers to retentivity while OR refers to coercivity, for permanents both retentivity and coercivity should be high.